CREDIT RISK, A MACROECONOMIC MODEL APPLICATION FOR ROMANIA

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Introduction

In this study we apply a macroeconomic credit risk model which links a set of macroeconomic factors (GDP growth rate, real interest rate of credit institutions on loan, exchange rate on forex market RON/EUR, industry-specific indebtedness) and industry-specific corporate sector default rates (industry, services, construction, agriculture) using Romanian data over the time period from 2002:2 to 2007:2. We are following Virolainen's (2004) methodology, modelling and estimating industry-specific default rates which yields better loan loss estimates than models based on aggregate corporate sector default rates only. We simulate with Monte Carlo method a loss distribution over a one-year time horizon by using the estimated industry-specific default rates and a hypothetical credit portfolio. Finally we will analyze the impact of adverse developments in interest rate on loan on the hypothetical corporate credit portfolio loss distribution, as well as on the expected and unexpected loss.

1. The theoretical credit risk model

Empirical studies indicates that macroeconomic conditions are likely to impact all components of credit losses, i.e. the probability of default (PD), the loss given default (LGD) and the exposure at default (EAD). In this study, we concentrate on the relationship between macroeconomic development and probability of default (PD).

One of the few credit risk models that explicitly links macroeconomic factors and corporate sector default rates was developed by Wilson (1997a, 1997b). The model has been applied to Austrian data at the aggregate corporate sector level by Boss (2002). His findings suggest that industrial production, inflation, the stock index, the nominal short-term interest rate and the oil price are the most important determinants of corporate default rates. Virolainen (2004) applies Wilson's model to analyze industry-specific default rates for the Finnish corporate sector. Virolainen uses the following macroeconomic variables to determine the default rates: GDP, interest rate and the corporate sector indebtedness level (Benyovszki, Petru, 2007).

We also apply Wilson's model (1997a, 1997b) to analyse industry-specific default rates in Romania.

As a first step we start with the modelling of the average default rate for industry *i* by the logistic functional form[1] as:

$$p_{i,t} = \frac{1}{1+e^{y_{i,t}}}, \qquad (1)$$

where $p_{i,t}$ is the default rate in industry *i* at time *t*, $y_{i,t}$ is the industry-specific macroeconomic index, whose parameters will be estimated, *i*, $i=\overline{1,m}$ indicates the number of industries.

[1] which is widely used in modeling bankruptcies to ensure that default rate estimates fall in the range (0,1).

We adopt Wilson's original formula and model the macroeconomic index in such a way that a higher value for $y_{i,t}$ implies a better state of the economy with a lower default rate $p_{i,t}$. Thus we obtain that:

$$L(p_{i,t}) = \ln\left(\frac{1-p_{i,t}}{p_{i,t}}\right) = y_{i,t}. \qquad (2)$$

The logit transformed default rate is assumed to be determined by a number of exogenous macroeconomic factors, i.e.:

$$y_{i,t} = a_{i,0} + a_{i,1} \cdot x_{1,t} + ... + a_{i,n} \cdot x_{n,t} + \mu_{i,t} \quad (3)$$

where a_i is a set of regression coefficients to be estimated for the i^{th} industry, $x_{j,t}$ is a set of explanatory macroeconomic factors (e.g. GDP, interest rate, etc.), in t period, $\left(j = \overline{1,n}\right)$ and $\mu_{i,t}$ is a random error assumed to be independent and identically normally distributed, $\mu_{i,t} \sim N(0,\sigma_j)$ and $\mu_t \sim N(0,\Sigma_\mu)$, where μ_t indicates the array of error terms $\mu_{i,t}$ and Σ_μ is its variance-covariance matrix. The equations (1) and (3) can be seen as a multifactor model for determining industry-specific average default rates. The systemic component is captured by the macroeconomic variables $x_{j,t}$ with an industry-specific surprise captured by the error term $\mu_{i,t}$.

Follows the second step, where we model and estimate the development of the individual macroeconomic time series. We use a set of univariate autoregressive equations of order n (*AR(n)*):

$$x_{j,t} = b_{j,0} + b_{j,1} \cdot x_{j,t-1} + ... + b_{j,n} \cdot x_{j,t-n} + \varepsilon_{j,t} \quad (4)$$

where b_j is a set of regression coefficients to be estimated for the j^{th} macroeconomic factor, $x_{j,t}$ indicates the value of macroeconomic factor j in the period t, and $\varepsilon_{j,t}$ is a random error assumed to be independent and identically normally distributed in t period, $\varepsilon_{j,t} \sim N(0,\sigma_j)$ and $\varepsilon_t \sim N(0,\Sigma_\varepsilon)$, where ε_t indicates the array of error terms $\varepsilon_{j,t}$ and Σ_ε is its variance-covariance matrix.

Equations (2)-(4) together define a system of equations governing the joint evolution of the industry-specific default rates and associated macroeconomic factors with a $(i+j)\times 1$ vector of error terms, E, and a $(i+j)\times(i+j)$ variance-covariance matrix of errors, Σ, defined by:

$$E = \begin{pmatrix} \mu \\ \varepsilon \end{pmatrix} \sim N(0,\Sigma), \quad \Sigma = \begin{bmatrix} \Sigma_\mu & \Sigma_{\mu,\varepsilon} \\ \Sigma_{\varepsilon,\mu} & \Sigma_\varepsilon \end{bmatrix}.$$

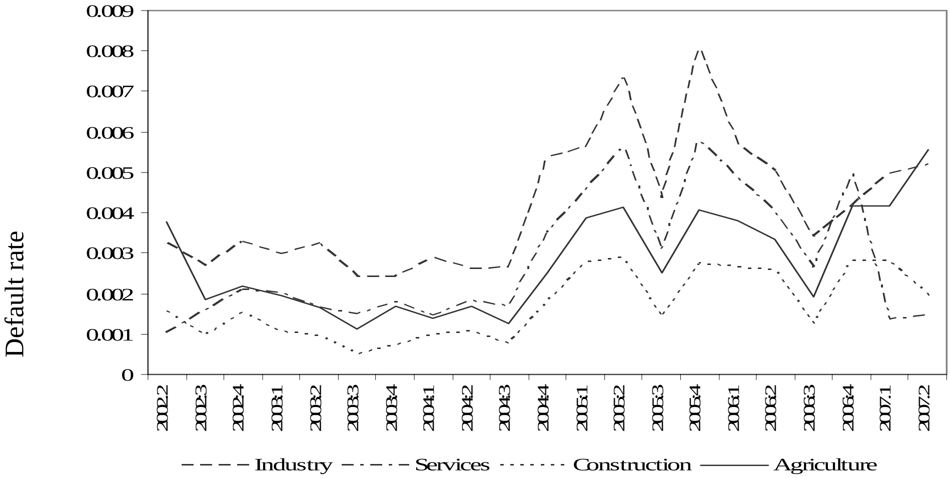
The final step is to utilize the parameter estimates and the error terms together with the system of equations to simulate future paths of joint default rates across all industries over some desired time horizon. By assuming that defaults are independent is possible to determine credit loss distribution for portfolios with Monte Carlo method.

2. The empirical application of the credit risk model

We apply quarterly data on corporate sector defaults by main industries and on macroeconomic factors over the time period from 2002:2 to 2007:2. We can obtain default rates by dividing the number of bankruptcy proceedings instituted by the number of active companies during the time period. We analyze the default data for the following four main industries according to the methodology used by the National Institute of Statistics: industry, construction, services (comprise activity of trade, transports, post and telecommunications, tourism, hotels and restaurants, general government and defence, education, health and social assistance and other services for economic units and for the population) and agriculture (comprise activity of agriculture, silviculture and pisciculture).

Figure 1 displays default rate by the four main industries over the sample period.

Figure 1: Quarterly default rates in the Romanian corporate sector by industries in period 2002:2 – 2007:2



Source: The National Trade Register Office

We studied the following specifications for the GDP variable: annual GDP growth rate, the deviation of GDP from trend and the GDP index[2]. Among these indicators the GDP index[3] has been chosen. We analyzed the explanatory power of the real interest rate on loan and the nominal interest rate on loan, among these the real interest rate *(r)* has the better explanatory power. For exchange rate quantification we have chosen the exchange rate on forex market RON/EUR *(Cv).* For corporate sector indebtedness *(C/VAB)* we use industry-specific variables, which are measured by the loan of an industry divided by the seasonally adjusted gross value added of that industry, each variable being presented at current prices.

We also analyzed the explanatory power of two additional variables: unemployment rate and consumer price index (CPI). The unemployment rate appeared to have the best additional explanatory power with significant coefficients, but strong correlation with the GDP variable.

We obtained the quarterly input data from the following sources:

- number of bankruptcy proceedings instituted, the number of active companies – The National Trade Register Office;
- real interest rate of credit institutions on loan, unemployment rate, exchange rate on forex market RON/EUR, volume of loans by industries – National Bank of Romania, Monthly Bulletins, 2002-2007 (www.bnr.ro);
- GDP index, GVA by industry, consumer price index – National Institute of Statistics, Monthly Statistical Bulletin, 2001-2007 (www.insse.ro).

According to empirical studies, we expect the GDP index to be positively related with the industry-specific macroeconomic indices, meanwhile the

[2] Volum index.
[3] The GDP index has been seasonally adjusted.

interest rate, exchange rate and the corporate indebtedness to be negatively related with them, since a higher value for the macroeconomic index implies a better state of the economy with lower corporate default rates.

We estimate the macroeconomic index equations for the four industries as static model with the seemingly unrelated regression (SUR) method in Gretl (Gnu Regression, Econometrics and Time-seriesLibrary, www.gretl.sourceforge.net).

Our results are presented in Table 1.

Table 1: SUR estimates for the static model (Sample period 2002:2-2007:2)

	Y_{IND}	Y_{SERV}	Y_{CONSTR}	Y_{AGR}
PIB	3.37075 (0.004131)***	4.57147 (0.000059)***	3.04515 (0.009872)***	3.86967 (0.000084)***
r	-0.928843 (0.000840)***	-0.595876 (0.009106)***	-1.02046 (0.000730)***	-0.732546 (0.000669)***
Cv	0.802491 (0.003235)***	0.617078 (0.014766)**	0.760506 (0.020998)**	0.286665 (0.203906)
C/VAB_i	-0.928161 (0.085275)*	-0.506947 (0.059000)*	-0.0558176 (0.508524)	-0.0510503 (0.454128)
Hansen-Sargan over-identification test: Chi-square (12)= 17.4309 with p-value p=0.134093				

*Note: C/VAB_i variable is industry-specific, t - statistics in parenthesis, ***, ** and * indicate significance level 1%, 5% and 10%.. Source: Own calculation in Gretl*

The GDP, the interest rate and the industry-specific measures of corporate indebtedness have the expected sign in all equations. The corporate indebtedness of construction and agriculture is not statistically significant. In case of the agriculture the explanation is that in Romanian agriculture the dominance of the small and unregistered family farms is characteristic (in 2006 only 4.16% [Own calculation based on the following sources:Romanian Statistical Yearbook, Chapter III., 2005, p. 18., 20.;Monthly Statistical Bulletin, 8/2007, p. 106. http://www.insse.ro/cms/files/statisticicomunicate/somaj/somaj_IV_06.pdf] of the agricultural employment is registered as people who receive wages) .

Table 2 presents the results of univariate autoregressive equations.

Table 2: Estimates for AR macro factor models

	PIB	R	CV	C/VAB_ IND	C/VAB _SERV	C/VAB _CONSTR	C/VAB _AGR
C	0.632 (0.0052***	-0.023 (0.055)*			-0.247 (0.072)*		-0.309 (0.0418)**
x_{t-1}	1.002 (0.0003)***	0.956 (<0.00001) ***	1.356 (0.00003) ***	0.835 (<0.00001)***			0.272 (0.0127)**
x_{t-2}	-0.596 (0.013)**		-0.472 (0.051)*				
x_{t-3}							0.221 (0.084)*
x_{t-4}					1.273 (<0.00001)***	1.153 (<0.00001) ***	1.369 (<0.00001) ***
Adj. R^2	0.559	0.993	0.825	0.805	0.904	0.986	0.919
DW	2.325	1.512	2.331	1.866	1.900	1.182	1.698

*Note: t-statistics in parenthesis, ***, ** and * indicate significance level 1%, 5% and 10%.*
Source: Own calculation in Gretl

The adjusted R^2 indicates a good determination of the dependent variable by independent variables in almost all of the equations. The Durbin-Watson (DW) statistics indicate no significant autocorrelation in the data, because its values are near 2.

3. Simulation of credit loss distribution

With the estimated parameters and the system of equations (2)-(4), we can simulate future values of joint industry-specific default rates with Monte Carlo method. Assuming that default dependence is entirely due to common sensitivity to the macroeconomic factors, the simulation over one year time horizon will have the following steps.

First, the Cholesky decomposition of the variance-covariance matrix of the error terms Σ is defined as A, so that $\Sigma = A \cdot A'$. Second, for each step of the simulation an $(i+j)\times 1$ vector of standard normal random variables $Z_{t+s} \sim N(0,1)$ is drawn. This is transformed into a vector of correlated error terms in the macroeconomic factors and the industry-specific default rates by $E_{t+s} = A' \cdot Z_{t+s}$. Using the simulated realizations of the error terms and some initial values for the macroeconomic factors, the corresponding simulated values for $x_{j,t+s}$, $y_{i,t+s}$ and $p_{i,t+s}$ can than be derived using the system of equations (2)-(4). The procedure is iterated until the desired time horizon and the desired number of simulated path of default probabilities is reached.

The simulated path of future default rates can be used to determine loss distributions for hypothetical corporate credit portfolio. The defaults of individual debtors can be considered independent events and assuming further that the recovery rate is fixed, loss distributions can be determined under the assumption of binomially distributed defaults. The fix loss given default (LGD) parameter is assumed to be equal with 0.45[4] throughout the simulation.

We utilize a hypothetical credit portfolio consisting of 3,000 corporate loans. At the construction of the credit portfolio we considered the percentage distribution of corporate loans by industry. This was managed by taking the distribution of the value of loans (Industry 43.31%, Services 46.69%, Construction 7.14%, Agriculture 2.86%) and of the number of those companies which had credit applications (Industry 26.92%, Services 62.57%, Construction 6.82%, Agriculture 3.68%) by industry, based on data from National Bank of Romania. The total credit portfolio value is 100 million RON (Benyovszki, Petru, 2007).

The simulation of the credit loss distribution was made in Matlab with Monte Carlo method. One hundred thousand simulations have been made in similar conditions to determine the distribution of credit portfolio loss and its probability. Figure 2 presents simulated loss distribution for the defined credit portfolio over a one- year time horizon.

The resulting loss distribution is skewed to the right, as expected, due to the positive default correlation through joint sensitivity to the macro factors. The expectedloss ($EL = Exposure \times LGD \times PD$) of the hypothetical credit portfolio (conditional on the macroeconomic environment) equals 1.27% of the corporate credit exposure over a one-year horizon.

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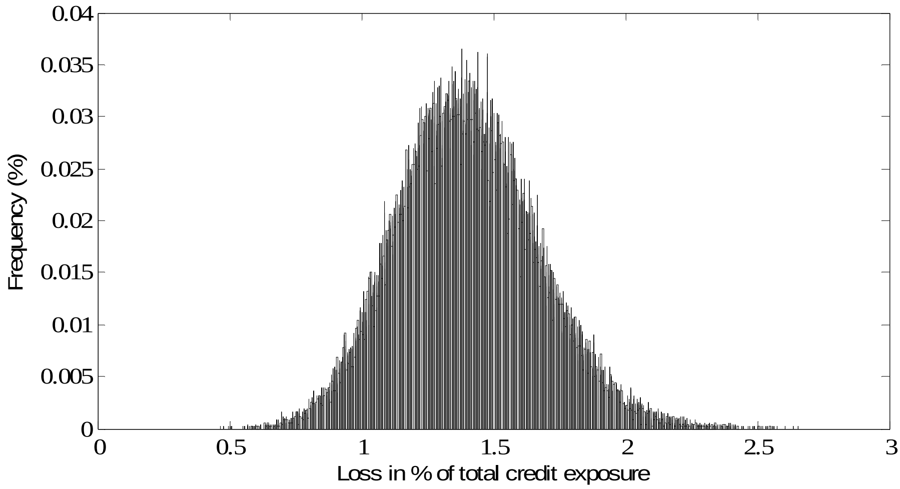
[4] Recomended by the new Basel Capital Accord (Basel II).

Figure 2: Simulated loss distribution of the hypothetical corporate credit portfolio in 1-year horizon



Source: Simulation in Matlab (100,000 simulations)

Unexpected losses are defined as the difference between the losses pertaining to the 99th and 99.9th percentile and the expected losses. The value of the unexpected loss is 2.48%, respectively 2.63% of total credit exposure. Expected and unexpected losses (for the 99th and 99.9th percentiles) for this loss distribution are represented in Table 3.

Table 3: Expected and unexpected losses of the hypothetical credit portfolio (in percent of total credit exposure, 1-year time horizon)

Expected loss	1.27%
Unexpected loss (VaR 99%)	2.48%
Unexpected loss (VaR 99.9%)	2.63%

Source: Own calculation in Matlab

4. Portfolio stress testing

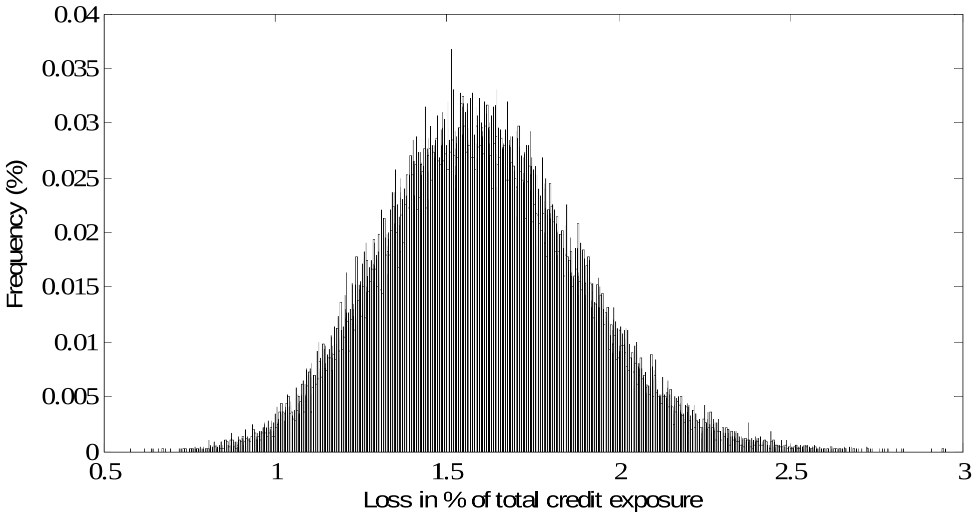
Stress test is an important tool in financial institutions' risk management, are used to complement financial institutions' internal model, such value-at-risk (VaR) models. Standard VaR models have been found to be of limited use in measuring financial institutions' exposure to extreme market events, i.e. events that occur too rarely to be captured by statistical models, which are normally based on relatively short periods of historical data.

An artificial shock can be introduced in the vector of error terms for stress testing purposes. The corresponding element in the vector $Z_{t+s} \sim N(0,1)$ of random numbers is replaced by the assumed shock. This shock is introduced in the first step of each simulation round and it has its impact to the other macro factors through the variance-covariance matrix. Loss distributions for the assumed stress scenario can than be calculated with the simulated default rates.

We assumed that for some exogenous reason the real interest rate increases by two percent for four consecutive quarter years. As result of this shock the default rates and the expected and unexpected losses will increase.

Similarly the above generated simulation we made 100.000 simulations to determine the credit portfolio loss distribution and its probability. The simulated loss distribution of the hypothetical credit portfolio over a one-year horizon is presented in Figure 3.

Figure 3: Simulated loss distribution of the hypothetical corporate credit portfolio in interest rate shock (increase) scenario, 1-year horizon



Source Simulation in Matlab (100,000 simulations)

Comparing the outcome with the initial results we can observe some increase in the expected loss and in the unexpected losses, because the relation between the interest rate and probability of default is direct. The expected loss of the portfolio increased from 1.27% to 1.44% of total credit exposure because of the interest rate increase. The unexpected loss (for the 99^{th} percentiles) increased from 2.48% to 2.59% of total credit exposure, meanwhile the unexpected loss for the 99.9^{th} percentiles increased from 2.63% to 2.71% of the total credit exposure (see Table 4.).

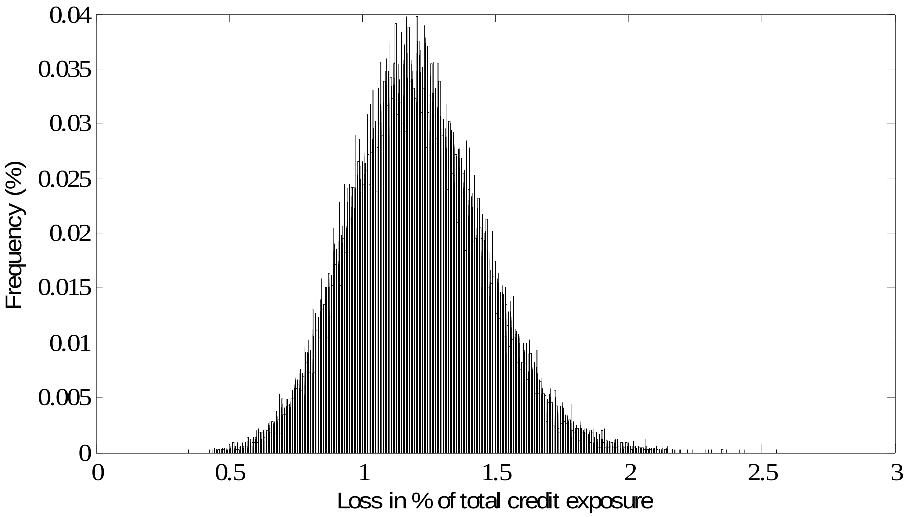
Table 4: Expected and unexpected losses of the hypothetical credit portfolio in the interest rate shock (increase) scenario (in percent of total credit exposure, 1-year time horizon)

Expected loss	1.44%
Unexpected loss (VaR 99%)	2.59%
Unexpected loss (VaR 99.9%)	2.71%

Source: Own calculation in Matlab

In the last phase a reverse shock was generated through an interest rate decrease with 200 bps in four consecutive quarters. The value of the other macroeconomic factors remained unchanged. The interest rate decrease implies the probability of default decrease, as well as the decrease of the expected and unexpected loss. The credit portfolio loss distribution is indicated by Figure 4.

Figure 4: Simulated loss distribution of the hypothetical corporate credit portfolio in interest rate shock (decrease) scenario, 1-year horizon



Source: Simulation in Matlab (100,000 simulations)

As the result of the interest rate decrease, the probability of default, the expected and the unexpected loss decreased. The expected loss caused by a better state of the economy decreased from 1.27% to 1.08% of the portfolio value as it can be seen in the Table 5.

Table 5: Expected and unexpected losses of the hypothetical credit portfolio in the interest rate shock (decrease) scenario (in percent of total credit exposure, 1-year time horizon)

Expected loss	1.08%
Unexpected loss (VaR 99%)	2.33%
Unexpected loss (VaR 99.9%)	2.52%

Source: Own calculation in Matlab

The main goal of this study is the determination of the expected and unexpected loss of the hypothetical credit distribution, the analysis of the changes of this loss caused by the changes of the macroeconomical environment.

Conclusions

In this study we applied an macroeconomic credit risk model which links a set of macroeconomic factors (GDP growth rate, real interest rate of credit institutions on loan, exchange rate on forex market RON/EUR, industry-specific indebtedness) and industry-specific corporate sector default rates (industry, services, construction, agriculture) using Romanian data over the time period from 2002:2 to 2007:2. We are following Virolainen's methodology, modelling and estimating industry-specific default rates. We simulate with Monte Carlo method a loss distribution over a one-year time horizon by using the estimated industry-specific default rates and a hypothetical credit portfolio. Finally we will analyze the impact of adverse developments (decrease/increase) in real interest rate on loan on the hypothetical corporate credit portfolio loss distribution, as well as on the expected and unexpected loss.

As result of the interest rate increase by 200 bps in four consecutive quarters, the expected loss of the portfolio increased from 1.27% to 1.44% of total credit exposure because of the interest rate increase. The unexpected loss (for the 99th percentiles) increased from 2.48% to 2.59% of total credit exposure. As an impact of the reverse shock, the interest rate decrease by 200 bps in four consecutive quarters, the expected loss decreased to1.08% and the unexpected loss (for the 99th percentiles) to 2.33% of the total credit exposure.

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