MODELING RISK USING ELEMENTS OF GAME THEORY AND FRACTALS

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1. Introduction

The model is built starting from the principle of budgetary constraint of each individual, entity or project in the economy. The model is based on assumptions that any investment materialized through a project may support a cost in its budget endurance limit according to the maximum capacity of revenue generated. An investment achieved through a project involves access to a range of financial sources by paying a price, which generates a cost that will be supported up to maximum capacity of an investment to generate revenue. In other words "any investor will finance a project up to the level at which he is willing to lose"[1].

The budget constraint formula can be used as a tool in the analysis of alternatives for financing a project for the project manager, who must choose the optimal combination to support a specific project. Decision-maker has to choose between five types of financing sources that are compared with each other depending on the additional cost each one generates over the financial sustainability of a project. The purpose of the option analysis of funding sources is to identify the optimal combination of financing used to support a particular project inside the limits of cost sustainability by the revenues generated by the investment.

2. Model description

The subject, manager or business owner must choose the optimum funding source to finance a certain project. The funding sources used must be repaid from the project budget, which is limited by the investment capacity to generate revenue. The types of funding used to achieve a specific project investment are as follows: self-financing from revenues generated by the project, loan, equity financing, public-private partnership, non-refundable grant. These forms of financing can be combined to ensure a financially sustainable project and minimize the risk of imbalances in the projects' budget[2].

1. Function for budget constraint for a single financing alternative for project:

$$S_0(r_c + 1) = S_v$$, where:

- So is the initial amount invested (which is accessed through an internal or external source of financing a project);
- Sv is the reimbursed amount and it is composed of the cost and the principal of the loan;
- r is a cost rate calculated by the ratio of the total cost of financing these (commissions, interest, fees or other costs), specific to each type of financing and the amount borrowed. This rate is the standardization of cost for each type of funding.

[1] Trenca Ioan, (2005), Managementul financiar, Ed. Casa Cărţii de Ştiinţă,Cluj, p.16.

[2] Tulai Constantin, (2003), *Finanţele publice şi fiscalitatea,* Ed. Casa Cărţii de Ştiinţă, Cluj Napoca, p.83.

2. Function for budget constraint for a double financing alternatives for project:

$$S_x(r_x+1)+S_y(r_y+1)=S_v$$

$$S_x+S_y=S_0$$

where:

- So is the initial amount invested (which is accessed through an internal or external source of financing a project);

- Sv is the reimbursed amount and it is composed of the cost and the principal of the loan;

- r is a cost rate calculated by the ratio of the total cost of financing these (commissions, interest, fees or other costs), specific to each type of financing and the amount borrowed. This rate is the standardization of cost for each type of funding;

- x and y are indices that reflect the type of funding.

3. Function for budget constraint for a triple financing alternatives for project:

$$S_x(r_x+1)+S_y(r_y+1)+S_z(r_z+1)=S_v$$

$$S_x+S_y+S_z=S_0$$

The model assumes that the creation and implementation of a project implies the use of resources that generates certain results. Considering the financial aspects of project cash- flows, there are two distinct components of cost, namely: the operating cost and the financing costs. The cash flows related are split between revenues needed to cover the operating costs and those needed to cover the financing costs.

$$Chf \rightarrow S_{ch}$$

$$Cf \rightarrow S_v$$, where:

$$S_{ch}+S_v$$

- Cf refers to the financing costs;

- Chf refers to the operating cost of an investment;

- Sch refers to the amount of revenue necessary to cover operating cost;

- Sv is the amount of revenue necessary to cover the financing cost.

4. The budget constraint function can be written for a double financing alternatives for project, as follows:

$$S_x(r_x+1)+S_y(r_y+1)+Chf=S_v+S_{Ch}$$

$$S_x+S_y=S_0$$

$$Chf=S_{ch}$$

, where:

- So is the initial amount invested (which is accessed through an internal or external source of financing a project);

- Sv is the reimbursed amount and it is composed of the cost and the principal of the loan;

- r is a cost rate calculated by the ratio of the total cost of financing these (commissions, interest, fees or other costs), specific to each type of financing and the amount borrowed. This rate is the standardization of cost for each type of funding;

- x and y are indices that reflect the type of funding.

The formula in paragraph 4 can be rewritten for the budget constraint function for two alternative financing used to finance a project, by eliminating operating costs and revenues from both sides of the equation to balance the budget, as follows: $S_x(r_x+1)+S_y(r_y+1)=S_v$.Where the terms in the equation were detailed in the formula above.

But the model is built starting from the premises that there is a level (V) of revenue generated by the project which represents the maximum recovery of financing costs. There is the budget constraint maximum capacity of a project generate revenue, which depends on the internal systemic built up of the investment. A portion of this revenue, denoted by V is the maximum volume that is able to cover the financing cost.

We can derive the function of result from implementing and operating an investment as follows:

$f(x) = V - S_v$, where three situations can occur:

a) In this case, the maximum capacity of revenue generated by the project covers all financing costs, thus realizing a ***gain*** through the operating of the investment realized through the project;

b) In this case, the maximum capacity of revenue generated by the project covers all financing cost, the project is at ***break-even*** level, in financial terms;

c) In this case, the maximum capacity of revenue generated by the project covers all financing costs, thus realizing ***losses*** through the operating of the investment realized through the project.

3. What criteria is used in the decision – making process?

Criteria used by the decision-maker to evaluating various alternatives for funding the project is the difference between project costs and revenues. In other words, the criteria for assessing the sources of funding relates to the gain or loss derived from the operating of the investment. A second criterion considered in the selection of appropriate funding source to achieve a particular project, is the financial risk.

Depending on the different tools to measuring risk, there are three forms currently used in economic theory:

- estimating probability of occurrence of a desired event (eg, a profit increase, an increase in the value of shares) or undesirable (eg the emergence of a financial loss, the lowering of the value of a stock dividend);
- evaluating each risky event with a score that could influence the development of project shows three categories of risk, such as: large, medium or low;
- estimating probability of achieving the "expected" levels of relevant financial performance indicators (Eg "expected" return for money invested in stocks).

Further on, the financial risk can take different forms due to the methods used for measuring it:

1. the risk of financial turbulence due to the systemic imbalances;
2. the risk of random events and their impact on the change of equilibrium level of financial indicators;
3. the risk of achieving expected-values associated with performance indicators of financial activity within a project.

In the first case, the equations on financial balance between revenue and expenditure are used. Profit or revenue level higher than cost are measured inside the equilibrium level.

In the second case, the method used refers to giving scores to random events based on subjective perception of individuals of future economic evolution.

In the third case, risk assessment is used especially in financial markets by estimating the expected return associated probability.

Risk modeling on the subjective probability of occurrence of risk concept for achieving financial balance according to the form of financing used is done by using the formula called "power law" of the theory of fractals[3] as follows:

$$N = M^d$$

- Where: N is the number of units of subjective probability of occurrence of risk;

- M is related the size of the unit used to quantify the probability of occurrence of risk;

- d is a coefficient of size-specific subjective probability of risk assessment.

Methodology of quantifying risk refers to estimating the perceived subjective probability at the individual level, through a mathematical formula, through

[3] Mandelbrot Benoit B, (1983), *The Fractal Geometry Of Nature, Updated anld Augmented*, International Business Machines Thomas J. Watson Research Center Freeman and Company, New York, p.10

induction and elements of game theory[4]. The formula of risk function (R (x)), as the subjective perception of risk at individual level is as follows:

$$R(x) = \begin{Bmatrix} \left(\frac{-1}{k} \bullet -x\right)^{\beta}, x < 0, \beta > 0 \\ \left(\frac{1}{k} \bullet x\right)^{\alpha}, x \geq 0, \alpha > 0 \end{Bmatrix},$$

where:

- 1 / k is a parameter project specific and it depends on intrinsec characteristics of the project. In the model, k is the amount borrowed (So);
- α, β, are the exponents describing a fractal[5] non-linear evolution of human-subjective phenomena;
- x refers to a unit of the result, which can be a loss (f (x) <0), a gain (f (x)> 0) or break-even point.

There are two risky situations:

a1.) If successful, f (x)> = 0, we have R (x)> = 1, reflects minimal risk to the financing costs not to be recovered from revenues generated by the project;

a2.) If successful, f (x)> = 0, we have R (x) <1, there is a risk that can grow exponentially in generating losses for the project, due to the incapacity of project revenues to cover full costs;

b1.) In case of losses, f (x) <0, we have R (x)> = 1, that reflects risk that can increase exponentially in generating losses for the project, the incapacity of revenues to cover the entire financing costs for the project;

b2.) In case of losses, f (x) <0, we have R (x) <1, there is a minimal risk, which may be diminished exponentially and reduce the level of losses for the project.

Risk function is defined as[6] having fractal characteristics due to human-subjective perception of risk of each individual due to information asymmetry and psychological factors of human perception[7].

From the perspective of classical finance, trigger factors of decision-action mechanism concerns the evaluation of the final values used to measure future economic transaction when compared with current values (discounted future values at an average discount rate are used for comparisons to present values). The concept of net present value is used for comparison between transactions at different points in time. This classical theory is based on the assumptions that a sum of money has a real purchasing present power and over a time it will have a different one due to the effect of the rate of inflation and the interest rate.

Recent theories such as cumulative prospect theory presents as an alternative to evaluate results of economic transactions through the concept of marginal value. Marginal values are the differences perceived in time between two different states or two similar goods (and interchangeable). The subject will select the option that provides the highest difference between all the alternatives available. According to this theory, individuals attach different meanings to a negative value (loss) compared with a positive one (gain). Therefore decision will take into account the context of decision, and that is the nature of the final outcome (loss or gain). Another difference in the existing theories concerning the value of options granted by the subject relates to the shape of this function. According to the theory of marginal utility, the function is linear in comparison with the value function described in the cumulative prospect

[4] Camerer Colin, Teck-Hua Ho, Juin Kuan Chong, (2001), *Behavioral Game Theory: Thinking, Building and Teaching,* Research paper NSF grant, p.24

[5] Lapidus M., Machiel van Frankenhuijsen, (2006), Fractal Geometry, Complex Dimensions and Zeta Functions Geometry and Spectra of Fractal Strings, Springer Science, Business Media, LLC pp.41-45.

[6] Taleb Nassim, Benoit Mandelbrot, (2005), Fat Tails, Asymmetric knowledge, and decision making. Nassim Nicholas Taleb"s Essay in honor of Benoit Mandelbrot"s 80th birthday, Wilmott Magazine, 2005, p.2

[7] Teoria prospectelor cumulate (1992) a căror fondatori sunt D. Kahneman şi Tversky,p.23

theory which takes the form of fractal "power law".

4. How does decision take place?

The function representing the ability of the individual to risk taking (F (x)), depending on context (loss or gain) explains how the economic decision regarding choice of funding sources takes place and it is managed according to game theory elements in the formula:

$$F(x)=\begin{Bmatrix} 1\Leftrightarrow f(x)\geq 0\wedge R(x)\geq 1 \\ -1\Leftrightarrow f(x)\geq 0\wedge R(x)<1 \\ 1\Leftrightarrow f(x)<0\wedge R(x)\geq 1 \\ -1\Leftrightarrow f(x)<0\wedge R(x)<1 \end{Bmatrix}$$

5. Conclusions

The decision model described through this paper provides an intuitive view on decision-making mechanism that underlies beneath the selection of funding sources related to an investment project. This model is going to be empirically tested for scientific validation.

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