DETERMINATION OF THE INTERNAL CESSION PRICE IN THE COMMERCIAL BANKS THROUGH MATHEMATICAL METHODS – A CASE STUDY

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The internal cession price of a commercial bank represents, in our view, the transfer rate at which the resources of the bank are transferred from one responsibility center to another. The price is merely conventional and it is determined based on certain management rules and not on the rules confirmed by the state of the market and having no impact on the general results of that particular commercial bank.

Here are several arguments in favor of the usefulness of the determination of the internal cession price in a commercial bank:

♦ *those in charge feel more motivated.* If internal cession prices are determined, managers of the responsibility centers can evaluate (their own) performance;

♦ *operation of the bank is improved.* The process is based on the idea that personal interest of the manager is conditioned by the results of his work and the convergence between the two will lead to the maximization of the efficiency of the responsibility center he manages. For it might happen that decisions of the manager of a certain responsibility center are benefic for that particular center, but have negative effects on the bank in general;

♦ *profitability can be evaluated.* The internal cession price allows the profitability of a responsibility center to be evaluated, at a certain extent. However

determination of the internal cession price contains also some subjective elements and as a consequence evaluation of the profitability of the centers based on these prices can be a permanent source of conflict.

In our view, another reason why the internal cession price has to be determined in a commercial bank is the *marginal resource* (the last resource acquired or sold in order to balance investment resources) which is defined as an excess from the point-of-view of the releasing responsibility center and a deficit from that of the receiving responsibility center.

Within a responsibility center of a bank the above-defined marginal resource can usually be acquired *only* by redistribution of resources among the responsibility centers of the bank or from the responsibility center of the Treasury Management Department (the department which deals with external acquiring or investing the deficit, respectively the excess of resources of the bank in general, a department which is also called the *treasury pool*).

Dealing with the marginal resource compels us to introduce another two terms:

♦ *marginal cost*, the excess cost created by acquiring an additional resource. It is an addition to the class of total expenses expended due to the

increase of a certain volume of resources;

♦ *marginal income*, the excess income gained from investing an additional resource.

In view of the above-mentioned the following association of terms can be performed regarding the responsibility centers of a bank.

Deficit resource center – marginal resource to be acquired – marginal cost;

Excess resource center – marginal resource to be sold – marginal income;

Relevant literature suggests several views regarding the determination of the internal cession price, one of these being the usage of mathematical models for such a purpose.

The case study hereby is based on such a view.

In the case study hereby the budgeted and actual excess and deficit of resources of 6 (six) profit centers of a commercial bank at the end of a calendar month will be presented using hypothetical data due to practical considerations.

Each of the 6 (six) profit centers represents a typical position in which any other profit center of the same bank can find itself.

1. The excess and deficit achieved by the 6 (six) profit centers compared to the budgeted values are presented in Table 1.

Table 1. The excess and deficit achieved by the studied profit centers at the end of a calculation period

lei or *thousands of lei* or *millions of lei*

Profit centers	Budgeted results		Actual results		Disparity	
	Excess	**Deficit**	**Excess**	**Deficit**	**Excess**	**Deficit**
0	1	2	3	4	5 = 3-1 or 3+2	6 = 4-2 or 4+1
PC1	+100	-	+150	-	+50 (E1)	-
PC2	+125	-	+70	-	-55 (E2)	-
PC3	+50	-	-	45 (95)		95 (D3)
PC4	-	110	-	100	-	-10 (D4)
PC5	-	70	-	95	-	25 (D5)
PC6	-	95	+20 (115)	-	+115 (E6)	-
Total	**275**	**275**	**240**	**240**		

2. Formation of the coefficients' matrix (A). This will be formed automatically by the computer program and it will be communicated to the local managers at the beginning of the evaluated period so that each manager would be aware of the internal cession price he will be allowed according to the results, i.e. the excess or deficit achieved compared to the budgeted values, at the very beginning of the process.

This coefficients' matrix may reflect the general policy of the bank; possible positive or negative deviations (not dependent on the local management policy) compared to the parameters determined at the beginning of the process will be reflected in the profit center of the General Management Department (Treasury Management Department – *treasury pool*), which connects the external environment of the bank and its internal profit centers. The Treasury Management Department is directly controlled by the general management of the bank and in our view, its results will reflect only its own performance (the general management's performance) without reflecting performances of local managements.

In our view, performances of local management should not be affected by factors outside their own activity. *The bank's general policy is formulated by the general management, but it is carried out by the local management.*

Each profit center (PC) will be assigned the following coefficients, according to its position in disparities regarding excess (E) and deficit (D), as follows:

Disp. E, D (thousands, millions etc.)

	+E	-D	-E	+D
0-20	5	5	5	1
21-40	4	4	5	1
41-60	3	3	5	1
61-80	2	2	5	1
81-100	1	1	5	1

One variant for formation of the coefficients' matrix is the one presented in the following logical scheme (logical scheme no. 1).

The actual method of determining the coefficients is applied according to the general policy of the bank, at the beginning of the financial year; therefore the coefficients determined at the beginning of a financial year cannot be changed during this interval.

Based on the conditions established above the coefficients' matrix (A) is as follows:

$$A=\begin{matrix} & CP1 & CP2 & CP3 & CP4 & CP5 & CP6 & \\ & 3 & -1 & 0 & 0 & 0 & 1 & E1 \\ & 3 & 5 & 0 & 0 & 0 & 1 & E2 \\ & 0 & 0 & 5 & 5 & -5 & 0 & D3 \\ & 0 & 0 & 5 & 5 & 5 & 0 & D4 \\ & 0 & 0 & 5 & -5 & 1 & 0 & D5 \\ & 3 & -5 & 0 & 0 & 0 & 1 & E6 \end{matrix}$$

Significance of the coefficients: e.g. +3 in the first row and first column signifies that three units of the excess achieved in PC1 will be assigned to PC1; +1 in the first row and last column (column 6) signifies that one unit of the excess achieved by PC1 will be assigned to PC6. The coefficients' matrix (A) will be formulated automatically by the computer program created for this purpose so that all classification errors regarding PCs will be excluded, they will be classified according to the Excess or Deficit achieved compared to the budgeted values.

3. *Restrictions in the system* are as follows:

$X1, X2, X3, X4, X5 \geq 0$ AND $30 \leq X6 \geq 40$

4. *The function of the problem* is maximization of the profit obtained by the means of the excess and deficit **achieved** by each PC and can be formulated as follows:

$Fx = 150X1 + 70X2 - 45X3 - 100X4 - 95X5 + 20X6 \longrightarrow$ maximum

5. *The column of the absolute terms (B)* is made up of the excess or deficit **budgeted** by each PC, as follows:

100;
125;
50;
110;
70;
95;

6. *The absolute term d = 0, the term in which we will be able to find (after solving the mathematical model using the simple reduction algorithm) the maximum of the function proposed by us (i.e. the maximum of the profit obtained by the means of the excess and deficit of resources);*

The mathematical model of the problem of lineal programming will have the following formulation:

$3X1 - X2 + 0X3 + 0X4 + 0X5 + 1X6 \geq 100$ budgeted excess to be maximized;
$3X1 + 5X2 + 0X3 + 0X4 + 0X5 + 1X6 \geq 125$ budgeted excess to be maximized;
$0X1 + 0X2 + 5X3 + 5X4 - 5X5 + 0X6 \geq 50$ budgeted excess to be maximized;
$0X1 + 0X2 + 5X3 + 5X4 + 5X5 + 0X6 \leq 110$ budgeted deficit to be minimized;
$0X1 + 0X2 + 5X3 - 5X4 + 1X5 + 0X6 \leq 70$ budgeted deficit to be minimized;
$3X1 - 5X2 + 0X3 + 0X4 + 0X5 + 1X6 \leq 95$ budgeted deficit to be minimized;
$0X1 + 0X2 + 0X3 + 0X4 + 0X5 + X6 \geq 30$
$0X1 + 0X2 + 0X3 + 0X4 + 0X5 + X6 \leq 40$

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Scheme no. 1. **Logical scheme of formation of coefficients' matrix A related to the above-defined problem**
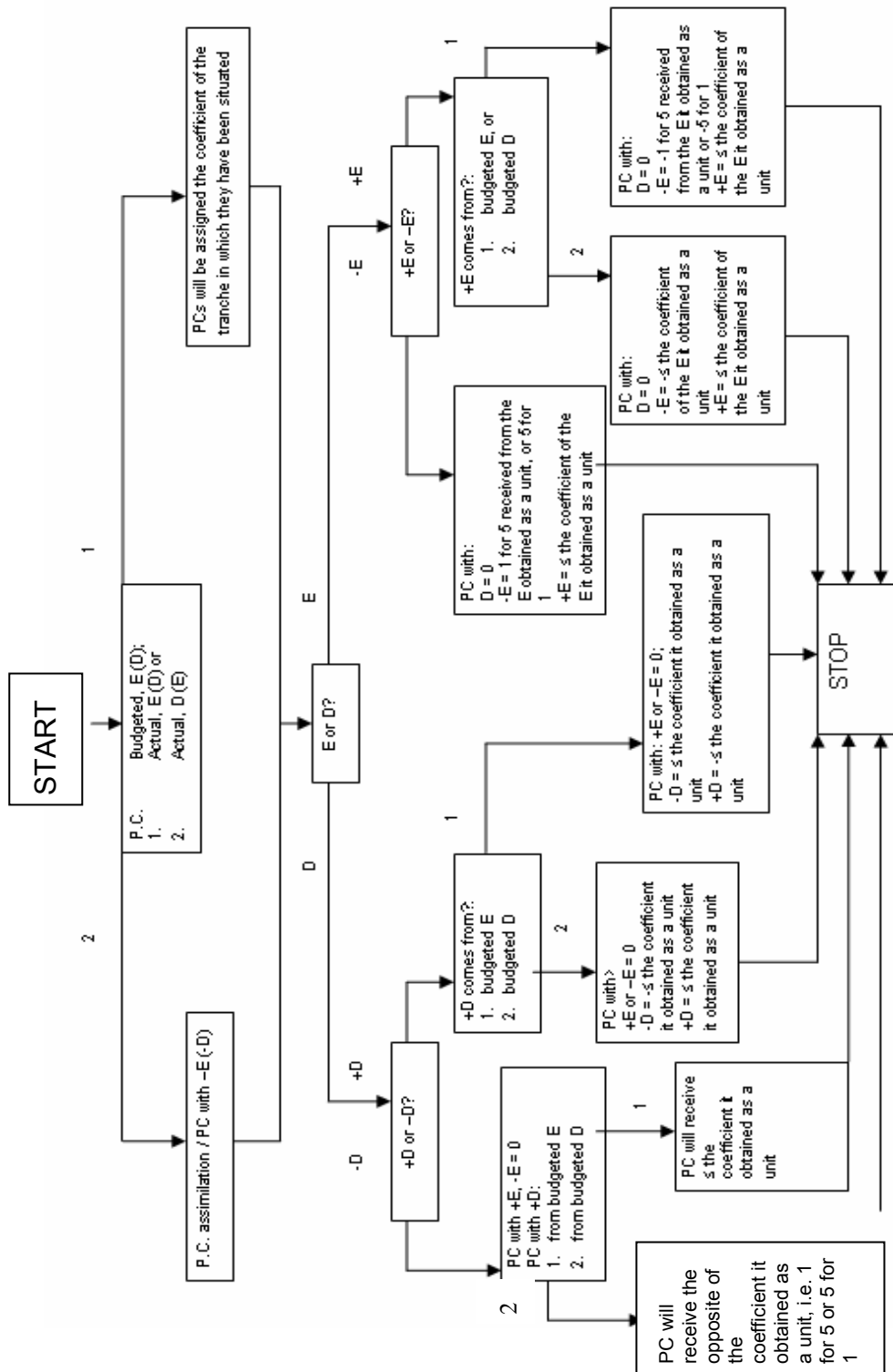
In accordance with the laws of lineal programming, in order that the excess or deficit of resources can be allotted in round numbers the inequalities above have to be transformed into equalities and thus the problem will be the following:

$3X1 - X2 + 0X3 + 0X4 + 0X5 + 1X6 - X7 = 100$
$3X1 + 5X2 + 0X3 + 0X4 + 0X5 + 1X6 - X8 = 125$
$0X1 + 0X2 + 5X3 + 5X4 - 5X5 + 0X6 - X9 = 50$
$0X1 + 0X2 + 5X3 + 5X4 + 5X5 + 0X6 + X10 = 110$
$0X1 + 0X2 + 5X3 - 5X4 + 1X5 + 0X6 + X11 = 70$
$3X1 - 5X2 + 0X3 + 0X4 + 0X5 + 1X6 + X12 = 95$
$0X1 + 0X2 + 0X3 + 0X4 + 0X5 + X6 - X13 = 30$
$0X1 + 0X2 + 0X3 + 0X4 + 0X5 + X6 + X14 = 40$

X7 … X14 - are variables of compensation with irrelevant value.
The matrix of the lineal programming problem will be as follows:

CP1 CP2 CP3 CP4 CP5 CP6

3	−1	0	0	0	1	−1	0	0	0	0	0	0	0	100	
3	5	0	0	0	1	0	−1	0	0	0	0	0	0	125	E2
0	0	5	5	−5	0	0	0	−1	0	0	0	0	0	50	D3
0	0	5	5	5	0	0	0	0	1	0	0	0	0	110	D4
0	0	5	−5	1	0	0	0	0	0	1	0	0	0	70	D5
3	−5	0	0	0	1	0	0	0	0	0	1	0	0	95	E6
0	0	0	0	0	1	0	0	0	0	0	0	−1	0	30	
0	0	0	0	0	1	0	0	0	0	0	0	0	1	40	
150	70	−45	−100	−95	20	0	0	0	0	0	0	0	0	0	

Applying the method of the simple reduction algorithm on the matrix of the linear programming problem using the *GIM-2008* computer program, the 3rd (third) step of the simple reduction algorithm shows the linear programming problem to have a solution in the aggregate of the positive numbers, which means the restriction system is compatible and the 6th (sixth) step offers the following results:

a). *Fx (the maximum of the function) = 6392,000;*

b). *The values of the unknowns are as follows:* $X1 = 26,667$;
$X2 = 3,000$;
$X3 = 14,400$;
$X4 = 1,600$;
$X5 = 6,000$;

$X6 = 30,000.$

If the 3rd (third) step had not allowed for a solution, we could have concluded that the lineal programming problem does not allow for a solution in the aggregate of the positive numbers. In this case the restriction system would have proven incompatible.

7. Analysis of the results obtained compared to the initial data

Comparing the excess and deficit of the studied profit centers and having in view the results obtained by solving the lineal programming problem defined above the data summarized in Table 13 have been obtained.

Table no. 2. The excess and deficit achieved by the studied profit centers compared to the solutions obtained by solving the lineal programming problem defined above

Profit centers (PC)	Budgeted results		Actual results		Disparity		*Value of the unknowns.*
	Excess (E)	Deficit (D)	Excess (E)	Deficit (D)	Excess (E)	Deficit (D)	
0	1	2	3	4	5 = 3-1 or 3+2	6 = 4-2 or 4+1	7
PC1	+100	-	+150	-	+50 (E1)	-	*26,667 income*
PC2	+125	-	+70	-	-55 (E2)	-	*3,000 income*
PC3	+50	-	-	45 (95)		95 (D3)	*14,400 cost*
PC4	-	110	-	100	-	-10 (D4)	*1,600 cost*
PC5	-	70	-	95	-	25 (D5)	*6,000 cost*
PC6	-	95	+20 (115)	-	+115 (E6)	-	*30,000 income*
Total	275	275	240	240			
Maximum of the function (the profit achieved from the excess and deficit of the resources of the PC)							*6392,000*

Comparing the initial data with the results obtained by solving the mathematical model the following can be concluded:

1) the internal cession price obtained is different both for the profit centers achieving an excess of resources and for those achieving a deficit, according to the discrepancy between the actual and the budgeted achievements;

2) profit center no. 1 (PC1) who exceeded its budgeted resources with 50% will receive a cession price equaling its income, more than PC2 (who did not achieve its budgeted resources), but less than PC6 (who exceeded its budgeted resources although it budgeted for a deficit);

3) profit center no. 3 (PC3) will receive the highest cession price equaling costs, since it budgeted an excess of resources and achieved instead a deficit, while PC4 will receive the lowest internal cession price equaling costs, because it achieved an even lower deficit than it budgeted. PC5 achieved a higher internal cession price equaling costs than PC4, since it did not achieve a lower deficit than it budgeted and thus caused additional costs to the bank, but still its internal cession price is much lower than that of PC3 who budgeted an excess of resources and achieved instead a deficit;

4) the internal cession price determined with the above method helps the bank to redistribute the effect in a more correct way, according to the performance of each unit and stimulates the profit centers to work for achieving the budgeted parameters or to exceed them or to achieve a lower deficit than budgeted - all natural aspirations for any management of any bank.

REFERENCES

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