A GEOMETRICAL MODEL OF DIRECT CONNECTIONS BETWEEN THE ECONOMIC PHENOMENA

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Abstract: In this paper we approach the issue of the evolution of economic phenomena that influence one another. First of all, we introduce a geometric model to establish whether there is a direct influence between two economic phenomena. After this, we find out the strength of this influence and, finally, we suggest a model for the form of this influence.

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1. Introduction

The activity of the economic agents represents one of the most complicated phenomena of humanity. The environment, in which this activity takes place, is a very complex one, made of numerous phenomena, which interacts one another.

Thus, in order to study an economic phenomenon, one needs to consider it together with all the connected phenomena.

The analysis of connections between economic phenomena is very important because, "true and lasting progress will be made only when we no longer regard the objects of our scientific observations merely as unrelated occurrences, but attempt to discover their causal connections and the laws to which they are subject" (Menger Carl).

The analysis of behavior of economic phenomena is also necessary because "in case of complicated connections between the economic phenomena, the theoretical analysis enables more exact and more accurate decisions in practice" (Guzowska Małgorzata, 2005).

2. Analysis of direct connexions among economic phenomena

Every analysis of the evolution of an economic phenomenon needs to go through the following steps: first is to establish whether or not there are any connexions between the studied phenomenon and other economic phenomena, and in case the connexions exist, their form and strength need to be determined.

By the strength of a connexion, we understand how strong, the evolution of the studied phenomenon, influence the evolution of the other phenomena.

In the study of these connexions we will consider only the phenomena which interact directly, not those which interact indirectly, via other phenomena.

2.1 Establishing the existence of connexions between two phenomena

In order to study the evolution of an economic phenomenon, conventionally named *F*, we will estimate that its evolution is described, from the mathematical point of view, by a real function, *f*. We will call this function *the function of evolution of the phenomenon*. As the economic phenomena occur in time, we will consider that this function has time as its argument.

So by *f(t)* we want to say a value taken by the evolution function of phenomenon F, at the time *t*.

Remark. *Because the function of evolution measures the evolution of an economic phenomenon, so that it exists, then we can deduce that* $f(t) > 0$, *whatever* $t > 0$.

By considering time as an argument for the function of evolution of phenomena, we can obtain a bi-univocal correspondence between the various values of the evolution functions of the economic phenomena.

In our paper, for ease of expression we use, when there is no danger of confusion, the notion of evolution, instead of function of evolution and reciprocally.

The evolution of some economic phenomenon is characterized by 3 states, which are:

- an increasing state;
- an involution state;
- a stagnation state.

The increasing state is also known as the development state or growth state, and the involution state is the decreasing state or regression state.

We say that an economic phenomenon is going through an increasing state if this experiments a continuous growth.

We say that an economic phenomenon is going through an involution state, if its evolution, in time, leads to its disappearance.

We say that an economic phenomenon is going through a stagnation state, if its evolution, in time, is constant.

But let us also express these states from a mathematical point of view:

For this purpose, we'll consider the evolution of the relative phenomenon during a time interval [a,b].

Definition 1.1. We'll call that phenomenon F is going through a development state throughout the time interval [a,b] if, for any two moments in time, t_1 and t_2, with $t_1 < t_2$, we have $f(t_1) < f(t_2)$.

Definition 1.2. We'll call that phenomenon F is going through an involution state throughout the time interval [a,b] if, for any two moments in time, t_1 and t_2, with $t_1 < t_2$, we have $f(t_1) > f(t_2)$.

Definition 1.3. We'll call that phenomenon F is going through a stagnation state, throughout the time interval [a,b] if, for any two moments in time, t_1 and t_2, with $t_1 < t_2$, we have $f(t_1) = f(t_2)$.

Considering now any two economic phenomena, F and G, described by their evolution functions, f and respectively g, we can establish that their evolution can take place, either in the same direction (development, involution or stagnation), or in opposite directions, or indifferently.

Definition 1.4. The evolution of phenomena F and G goes in the same direction if the two phenomena are in the same evolution state.

Definition 1.5. The evolution of phenomena F and G goes in opposite directions, if one of the two phenomena is in one evolution state, for example, development state, and the other is in another evolution state, for example, involution.

Definition 1.6. If between the evolution of the two phenomena F and G there is no connexion, then their evolution is indifferent.

Definition 1.7. If there is no connexion between the two phenomena, then they can go through any of the three states.

If we analyze any two economic phenomena, we observe that they can be connected, or un-connected, or, during some pass of time, their connection can undergo changes.

Further on, we are going to define these things from a mathematical point of view and eventually, come up with a modality of establishing the fact whether there is or there is not a direct connexion between certain two economic phenomena and also we are going to find out the strength of the respective connexion.

Firstly we assume following supposition and this will have a major significance for the following theory. If the evolution of an economic phenomenon is constant in time, then the phenomenon will not influence the evolution of any other phenomenon, nor will its evolution be influenced by other phenomena's evolution.

Thus the only phenomena with interest for our theory, are those in either an increasing or decreasing state.

Definition 1.8. We say that the evolution of the economic phenomenon G influences directly the evolution of the economic phenomenon F, on the time interval [a,b] if for any three values g_1, g_2, g_3 taken by the evolution function of the phenomenon G, there are three corresponding values, f_1, f_2, f_3 taken by the evolution function of the phenomenon F, and between these values one of the following relations are established:

a)$g_1 < g_2 < g_3$ and 1) $f_1 < f_2 < f_3$, or 2) $f_1 > f_2 > f_3$. or

b)$g_1 > g_2 > g_3$ and 1) $f_1 < f_2 < f_3$ or 2) $f_1 > f_2 > f_3$.

Definition 1.9. We say that the evolution of the economic phenomenon G does not influence the evolution of the economic phenomenon F on the time interval [a,b], if between any three values g1, g2, g3 taken by the evolution function of the phenomenon G, and the three corresponding values f1, f2, f3 taken by the evolution function of the phenomenon F none of the above mentioned relations could be established.

If between the evolution of the economic phenomena G and F there is a direct influence, then:

- in the case a.1.: we have a direct influence of type increasing-increasing;
- in the case a.2.: we have a direct influence of type increasing-involution;
- in the case b.1: we have a direct influence of type involution-increasing;
- in the case b.2.: we have a direct influence of type involution-involution;

In other words, in the case a.1, both phenomena evolve in the same direction, that is, it is an increasing-increasing type of evolution. The development state of phenomenon G brings about a development state for phenomenon F. In the case a.2, the two phenomena evolve in opposite directions, that is, it is an increasing-involution type of evolution. The development state of phenomenon G brings about the disappearance of phenomenon F.

Cases b.1 and b.2 are interpreted in a similar manner. Specifically, in the case b.1, the two phenomena evolve in opposite directions, that is, it is an involution-increasing type of evolution. The disappearance of phenomenon G brings about a development state for phenomenon F, and in the case b.2, the evolution of both phenomena goes in the same direction, that is, it is an involution-involution type of evolution. The disappearance of phenomenon G brings about the disappearance of phenomenon F.

Example 1.1. Now we are able to give a few examples of phenomena that interact directly with each other. We assume that G represents the demand for cosmetical products and F represents the supply for these products. From everyday experience, we know that the evolution of phenomenon G influences the evolution of phenomenon F. In other words, the higher the demand is, the higher the supply has to be. The lower the demand is, the lower the supply will be.

So, in the two cases above, we have a direct increasing-increasing influence and a direct involution-involution influence from G to F.

Example 1.2. Let us now consider two other economic phenomena: let G be the job supply and F be the unemployment level. From everyday experience we also know that, if the supply of open jobs is high, then unemployment level is low and a low level of job supply leads to increased unemployment level.

Consequently between the two phenomena there is a growth-involution state or an involution-growth influence from G to F.

Generally, in the economic field, the influences between phenomena are not in only one direction, but the phenomena influence each other. If the evolution of an economic phenomenon G influences the evolution of another economic phenomenon F then the evolution of the phenomenon F is most likely to also influence the evolution of phenomenon G.

In order to exemplify these ideas, we may consider any of the examples given before, but we may also give another one.

Example 1.3. Let us assume that phenomenon G is represented by the economic development of one region, and phenomenon F is represented by the over-trained labor force available at one time.

If the economic development in one region is low than there is no need for over-trained labor force; instead, if there is over-trained labor force available then this can lead to the economic development of the respective region. If a region is economically highly-developed, than there will be need for over-trained labor force, but the lack of over-trained labor force available will lead to economic decrease in the respective region.

Thus, everything we are about to say is completely justified.

Similar definitions to the ones provided for phenomena G and F can be given also for the direct influence of the evolution of the economic phenomenon F over the evolution of phenomenon G.

If the evolutions of both economic phenomena G and F reciprocally influence directly each other then we may say that the two phenomena interact directly, that is they are in direct connexion or linked.

In other words, between two economic phenomena, F and G, there is a direct connexion, if the evolution of one phenomenon influences the evolution of the other one.

Definition 1.10. So, if between two phenomena, F and G, there is a direct connexion, that is, they interact directly, then between the values f_1, f_2, f_3 taken by the evolution function of the phenomenon F and the corresponding values g_1, g_2, g_3 taken by the evolution function of the phenomenon G, one of the following relations can take place:

i) $g_1 < g_2 < g_3$, and 1) $f_1 < f_2 < f_3$, or 2) $f_1 > f_2 > f_3$ or

ii) $g_1 > g_2 > g_3$, and 1) $f_1 < f_2 < f_3$, or 2) $f_1 > f_2 > f_3$.

In the cases i.1 and ii.2, both phenomena undergo an evolution in the same direction and in the cases i.2 and ii.1, both phenomena undergo an evolution in opposite directions.

2.2 Establishing the strength of direct connexions

Another issue to discuss regarding the study of the connexions between phenomena refers to the strength of influences that is how strongly the evolution of one phenomenon influences the evolution of the other one.

In order to measure the strength of these influences we will resort to a geometrical approach that is the slope of a straight line which passes through two given points. We take it as our approach because regardless of the evolution of a phenomenon, its graphic representation, or at least on short time intervals, can be approximated by a straight line.

As it is known, the slope of a straight line which passes through the points (t_1,g_1) and (t_2,g_2), noted by m, is given by the relation (1):

$$m = \frac{g_2 - g_1}{t_2 - t_1}$$

It is also known that if $m > 0$ then the respective phenomenon is going through a development state and, respectively, if $m < 0$ then it is going through an involution state.

We call that if a phenomenon F evolves constantly on a time interval $[a,b]$, (growing or decreasing), for any given three values f_1, f_2, f_3 taken by the evolution function the slopes of the straight line that passes through the points $((t_1,f_1), (t_2,f_2))$ and $((t_2,f_2), (t_3,f_3))$ are identical.

In order to measure the strength of the influence of the economic phenomenon G over phenomenon F, we will consider the slopes of the evolution straight lines:

$$m_g = \frac{g_2 - g_1}{t_2 - t_1} \quad \text{and} \quad m_f = \frac{f_2 - f_1}{t_2 - t_1}$$

Using these notations, we can obtain the following results:

a.1) if $m_g.m_f > 0$ then both phenomena evolve in the same direction. The development state of phenomenon G brings about a development state for phenomenon F and, reciprocally, the involution state of phenomenon G brings about an involution state for phenomenon F.

a.2) if $m_g.m_f < 0$ then the two phenomena evolve in opposite directions. The development state of phenomenon G brings about an involution state for phenomenon F and, reciprocally, an involution state of phenomenon G brings about a development state for phenomenon F.

If between two economic phenomena F and G, there is a direct connexion then, according to what we said before, we can infer the following, regarding the strength of the direct connexion between these two phenomena:

Let us consider the first case, a.1 when both phenomena evolve in the same direction. There are two sub cases, namely:

1.1) $m_g > 0$ and $m_f > 0$,

1.2) $m_g < 0$ and $m_f < 0$.

In the sub case of 1.1, both phenomena evolve in the same direction which is of the type increasing-increasing. The following situations can occur:

If $m_g > m_f$ then the growth of the phenomenon G is faster than the growth of the phenomenon F. The strength of the influence of the evolution of phenomenon G over the evolution of phenomenon F is low and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is high. In other words, a bigger increase in the evolution function of phenomenon G causes a smaller increase in the evolution function of phenomenon F, and reciprocally, a smaller increase in the evolution function of phenomenon F causes a bigger increase in the evolution function of phenomenon G.

If $m_g < m_f$ then the growth of phenomenon G is slower than the growth of the phenomenon F. The strength of the influence of the evolution of phenomenon G over the evolution of phenomenon F is high and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is low. In other words, a smaller increase in the evolution function of phenomenon G causes a bigger increase in the evolution function of phenomenon F, and reciprocally, a bigger increase in the evolution function of phenomenon F causes a smaller increase in the evolution function of phenomenon G.

Also in the sub case of 1.2, both phenomena evolve in the same direction, but in an involutive-involutive type of evolution. In this case, the following situations can occur:

If $|m_g| > |m_f|$ then the involution of phenomenon G is faster than the involution of phenomenon F. The strength of the influence of the evolution of phenomenon G over the evolution of phenomenon F is low and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is high. In other words, a bigger decrease in the evolution function of phenomenon G causes a smaller decrease in the evolution function of phenomenon F, and reciprocally, a smaller decrease in the evolution function of phenomenon F causes a bigger decrease in the evolution function of phenomenon G.

If $|m_g| < |m_f|$ then the involution of phenomenon G is slower than the involution of phenomenon F. The strength of the influence of the evolution of phenomenon G over the evolution of phenomenon F is high and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is low. In other words, a smaller decrease in the evolution function of phenomenon G causes a bigger decrease in the evolution function of phenomenon F, and reciprocally, a bigger decrease in the evolution function of phenomenon F causes a smaller decrease in the evolution function of phenomenon G.

Considering the second case, a.2, when the two phenomena evolve in opposite directions, there will occur two sub cases, namely:

2.1) $m_g > 0$ and $m_f < 0$,

2.2) $m_g < 0$ and $m_f > 0$.

In the sub case of 2.1, then the phenomenon G undergoes a development state and the phenomenon F undergoes an involutive state. It means, that the evolution of these two phenomena is of type increasing-involution. Subsequently, the following possible situations will take place:

If mg > |mf| then the growth of phenomenon G is faster than the decrease of the phenomenon F. The strength of the influence of the evolution of phenomenon G over the evolution of phenomenon F is low and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is high. In other words, a bigger increase in the evolution function of phenomenon G causes a smaller decrease in the evolution function of phenomenon F, and reciprocally, a smaller decrease in the evolution function of phenomenon F causes a bigger increase in the evolution function of phenomenon G.

If mg < |mf| then the growth of the phenomenon G is slower than the decrease of the phenomenon F. The strength of the influence of the evolution of phenomenon G over the evolution of phenomenon F is high and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is low. In other words, a smaller increase in the evolution function of phenomenon G causes a bigger decrease in the evolution function of phenomenon F, and reciprocally, a bigger decrease in the evolution function of phenomenon F causes a smaller increase in the evolution function of phenomenon G.

In the sub case of 2.2, the phenomenon G undergoes an involutive state and the phenomenon F undergoes a development state. It means, that the evolution of these two phenomena is of type involution-increasing. Subsequently, the following possible situations will take place:

If $|m_g| > m_f$ then the involution of the phenomenon G is faster than the increasing of the phenomenon F. The strength of influence of the evolution of phenomenon G over the evolution of phenomenon F is low and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is high. In other words, a bigger decrease in the evolution function of phenomenon G causes a smaller growth in the evolution function of phenomenon F, and reciprocally, a smaller decrease in the evolution function of phenomenon F causes a bigger growth in the evolution function of phenomenon G.

If $|m_g| < m_f$ then the involution of the phenomenon G is slower than the increasing of the phenomenon F. The strength of the influence of the evolution of phenomenon G over the evolution of phenomenon F is high and the strength of the influence of the evolution of phenomenon F over the evolution of phenomenon G is low. In other words, a smaller decrease in the evolution function of phenomenon G causes a bigger growth in the evolution function of phenomenon F, and reciprocally, a bigger decrease in the evolution function of phenomenon F causes a smaller growth in the evolution function of phenomenon G.

Another namely the one when ,case can be mentioned$|m_g| = |m_f|$. In this case, the force of direct connection between both economic phenomena is moderate. The evolution of these two phenomena takes place in a parallel fashion. In that case, it is difficult to decide if there is any link between the two phenomena.

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