DYNAMICAL MODELING – AN INTERDISCIPLINARY APPROACH OF THE ECONOMIC REALITY

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Abstract: The present paper tries to explain the important role of mathematical modeling in the present society. The dynamical modeling can represent an instrument of strategy and prognosis on average term. It exist many dynamical models what can to offers information about the evolution in certain areas (the financier flow, the aggregate demand, the labor market) on average term. In the analyses dynamics macroeconomic area we can observe a big variety of method and techniques for research fluctuates from economy and financial date.

Key words: economical dynamics, economical equilibrium, dynamical modeling, time evolution.

1. ECONOMICAL EQUILIBRIUM, ECONOMICAL DYNAMICS, ECONOMICAL DEVELOPMENT, ECONOMIC GROWTH AND PROGRESS

Economic dynamics has three particular cases: economic development, economic growth and progress. The modern theory of economic development has first in view the economical balance corresponding to the equilibrium points of economical dynamic systems. Complete dynamic studies show that in economy the structural changes and oscillations are the rule and not the exception and the stationary statuses generally become unstable when certain parameters vary. This way, the economy may evolve to new economic cycles or to chaotic economical situations. This is what the economical dynamic study has imposed in replacement for the classical theories (static study) of economic balance, based more on the economical methods concerning algebra equations.

Economic development contains elements of success and failure. Generally, economists use the term **economic development** with a more meaningful sense, which incorporates cultural and institutional changes (Lewis). This way, there exists the capitalist and socialist economic development. Classical economists (Smith, Malthus, Ricardo, Marx, Mill) are interested in developing the social system, in one word the economic development which corresponds to economic evolution. In exchange, **economic growth** corresponds to the expansion and change of the economical balance. Economic development is associated to the instabilities and behavior of stocking while economic growth, stability and balanced expansion. **The progress** is an economical dynamic different in economic development and in economic growth. It concerns the perfecting and advance.

The key for surviving and growing is constituted by the capacity of a firm to adapt it's strategies with the environment permanently in change. This requires the management to anticipate correctly the future events. The anticipation is realized by the planning departments which develop long term previsions of the most important factors that affect the products market. Specialized firms realize long term previsions for different components of the macroeconomic medium like, for example, the economy, the population, natural resources and technology. The experts opinion, exploiting the trend, correlating the trend, econometric modulation, cross-impact analysis, request-hazard prevision also multiple scenarios are only some methodologies of realizing the macroeconomic previsions.

Previsions are made on the base of the dates and models and depend very much of these data. Economic models are powerfully influenced by the period in which the data was gathered and it's size. The modulation coasts in the fact that constructing a representation with a variable degree of fidelity of the real world or a component part of it. Understanding this phenomenon or its segment of reality took in consideration, knowing by detail also the action over the phenomenon that is analyzed motivates the ration for taking this kind of representations. The most used language is the mathematical language. Using the mathematical modulation helps the substantiation of a decision in the efficiency conditions, offering the possibility to think better and faster without denaturizing the reality. Mathematical formulating is not the only kind of abstracting form, but "it is the superior step of scientific abstract, which approaches us to the frontiers of certitude insofar as the premises are not falsified" after the T. Shattles firm.

Starting from the idea that any model is based on real parameters and data, it becomes necessary the fact that for obtaining trustful dates to permit a confinable representation of the reality by model. Therefore, when necessary, the cyclic or periodic aspect of the phenomenon studied is identified, implicitly the time horizon which is referred to.

Mathematising the economical disciplines has driven to crystallizing the micro and macroeconomic analysis, indifferent of the branch particularities, aborting common problems of any economy sector. The microeconomic is concerned as an isolated element of a bigger system, but not as something existing for itself. Economic systems are hierarchical synergetic systems, different economic dynamics corresponding to the level to which it is studied the economic phenomenon. The microeconomic models constitute the cores or "bricks" of which are composed macroeconomic models. Estimating the parameters of macroeconomic models in dependence of microeconomic characteristics is, actually, the main purpose of economic mathematics, like also other domains of quantitative economical domains, of high utility by the economic plans and politics point of view.

By economic system it is understood a system (primary notion) characterized by economic sizes. The economy is an analytic study which concerns of the existing relations or it can be supposed to exist between direct measurable sizes. The prices, mortgages, incomes, costs, quantity of sold product or bought by the market and quantities of factors of protection used by an enterprise are only some examples of variable sizes used in economy. Some of these sizes can be measured in natural or physical units, others only in money or other value units. Important for us here is that they are measured in some ordinary units. So there is no doubt that the mathematical methods can be applied in economy and that the economical relations may be expressed with the help of some

mathematical functions although sometimes economy conditions of the problem impose certain limits concerning the form of the functions, systems, etc.

Using econometric modulation researchers have realized equation systems which describe condition systems. Econometric models which contain more than 300 equations, for example, are used for premising the changes in American economy.

The activities in the economic space, for the individual economic agents or for an ensemble of economic agents which constitute in a bough or under of activity, or for their totality, at the level of national economic space, there can be analyzed, followed and optimized and by the application of some new modern methods, in a conception of aborting mathematics, in which the mathematical instrument plays a very important role. This kind of instrument is the theory of dynamic systems.

2. DYNAMICAL MODELING OF ECONOMIC PHENOMENON

In the '30s, Jan Tinberger began his research and elaborated the first macroeconomic methods with more equations. Their modern prototype, crystallized, is the Klein-Goldberger (1955) model which unites different economic variables – income, consume, taxes, spends, investments – by a round of macroeconomic variables – for example, functions or request.

Dynamical modeling was elaborated by J. Forrester in the period of the '60s and is based on the fact that the functioning of a system is represented by the knowledge of the interactions between the information fluxes, commands, human resources and material resources etc. A **dynamic model** surprises the behavior or complex systems showing how their structure determinates their trajectory, respectively their behavior in time.

In the economical dynamic there are met hundreds of mathematical models. They contain numerous parameters, this implies a high difficult apply and concerning the mathematics is reclaims using some topological methods of the singularities theory (catastrophes and bifurcations). While the parameters effects described by the global diagram of dynamic bifurcation is very different in different zones of the parameters space, finding the zones of structural stabilization and of inexistence of this stability (of bifurcation) is vital for the management of an economy.

The appearance of the theory of nonlinear dynamics has permitted the understanding and development of some processes and methods to approach us more to the phenomenon of reality. Developing the singularities theory and the bifurcation theory has completed the group of ways we dispose of for analyzing and representing more and more complex dynamics, giving the possibility of analyzing some systems which were hard, if not impossible to talk about by traditional methods. The study over nonlinear dynamics is of maximum interest because the economic systems are by excellence nonlinear systems. Many of these contain multiple discontinues and incorporate an intern instability being permanently submissive to the actions of the shocks, and intern and extern perturbations.

For low values of some parameters there can be produced extremely high changes of variables, so there can be produced bifurcations to subscribe the considered system for other trajectories.

The bifurcation theory has the advantage of a mathematical device very good elaborated, studying the existence and the stability of balance solutions, because a solution in instable balance cannot be observed in reality.

Because of the complexity of real processes, in the construction of methods it must be adopted a certain limit of detailing, restraining the essential elements and the main dependences between them. After that, the model must always be a simplified

representation of the reality to permit actions, based on rationality, over the modulated process.

Dynamic modulations follow distinguishing of temporary relations. The model operates with the events and settings which express the value of an attribute in which the events apparition is identified. With the help of data structures transition of states diagrams are built, these indicate the entire operations specific to every type of object and class corresponding.

Nowadays the necessity of dynamic systems study from micro and macro economy justify, along with the ones in biology, the rationality of development without precedent of the theory of the dynamic systems and its applications.

In economic dynamics, previsions are made on the base of the data from the models and depend very much on this data. In an important part these data, which are conform to the correct data registered in economic development, they are not made public. The economic models are powerfully influenced also by the period in which these data have been gathered and it's size.

Economical processes described by ordinary differential equations are named continuous processes. If the evolution is examined in discrete timing (year, month, trimester, semester, decade, week, day, hour) or by introducing a unity of stimulation, then the process dynamics will be described by discrete methods consisting of equations with infinite differences, equation systems with finite differences or recurrent equations.

The model "good", "stalwart", when the process structure on which is applied does not suffer serious changes at the data variant, so that the dynamic system corresponding is stabile structural For example, if the model foresees an evolution in a certain cyclic, than it's confirmation takes place only if in the system there has not been produced any structural mutations, like a massive intervention of the state which modifies the entire mechanism of reactions of the system. The problem of economic models is a problem of the type of dynamic process to which is applied in consequence of the data type series corresponding to the process.

The evolution of economical processes can be represented in different dynamic models concerning the complexity of the process, when a single main indicator y(t) is distinguished respectively an ensemble of indicators $y_1(t)$ … $y_n(t)$ correlated by the equations of the model both between them and with the factorial variables which condition the process $u_1(t)$ ... $u_m(t)$. The indicators are the unknown functions and represent the variables of state of the associated dynamic system. Treating the economic process as a system distinguishes the factors $u_1(t)$ ... $u_m(t)$ as entrances and, the indicators $\dot{y}_1(t) \ldots \dot{y}_n(t)$, $y_1(t) \ldots y_n(t)$, as variables of state of the system, outputs.

In a mathematical point of view, the factors are data.

We must remark that the economic process and phenomenon modulation method is, in present, a method of reference for the theory and practice of mathematic modulation. The model is built as an isoform representation of reality and offers an intuitive image but also rigorously to it in the sense of the logic structure of the studied phenomenon. This way, it is facilitated the discovery of some links and bindings which, practically, in other ways, would be impossible or very hard to find. The explanation of some evolutions is built on the results of such estimations.

1. Because of the complexity of real processes, in the constituting of models a certain detail limit must be adopted, restraining the essential elements and the principals

depending on them. That is why the model must be always a simplified representation of the reality which is to permit actions, based on rationality, over the modeled process.

3. A DYNAMICAL MODEL FOR EVOLUTION OF CAPITAL

For example, starting from the classic model multiplier-accelerator, we present from now on a model derived from this, the multiplier of the second grade. We suppose in the frame of this model a standard function for the investment, function which contains the difference between the expected capital (K^*) and the actual capital stock (K), meaning:

$$\dot{K} = I = \alpha \times (K^* - K)$$

Where $\alpha > 0$, represents the adjustment speed of the capital. For simplifying it's been considered that there exists no depreciation.

This equation represents the principle of adjustment of the capital stock. Forward, it is purposed $K^* = rY$, meaning that the stock of the proportional capital is the Y income, r being the rhythm of growth of the capital/income report. This way it is obtained:

$$\dot{K} = I = \alpha \times (rY - K)$$

By the second grade accelerator it is desired that following the capital stock development using to the actual level of investment I, but the wanted level I^*. Taking this in consideration we think that $\dot{I} = \beta(I^* - I)$, meaning:

$$\dot{I} = \beta \times (\alpha \times (K^* - K) - I)$$

The idea with the second grade accelerator is the result of the second process of decision: at first, the manager must follow the difference between the capital stock and the expected capital for making the necessary adjustment to the wanted level of investments (certain knowledge of the α value) and at second, it must follow the investment realization by the difference between the level expected an the actual level.

Because $\ddot{K} = \dot{I}$, realigning the terms it is obtained:

$$\ddot{K} + \beta \times \dot{K} + \alpha \times \beta \times K = \alpha \times \beta \times K^*$$

We construct now a macroeconomic model concerning of the standard function of consume without differences $C = a + b \times Y, 0 < b < 1$. By the equilibrium condition $Y = C + I$ there is obtained:

$$Y = \frac{1}{1-b} \times (I + a) = \frac{1}{1-b} \times (\dot{K} + a)$$

Replacing $K^* = r \times Y$ in the differenced equation of the second grade we obtain

$$\ddot{K} + \beta \times (1 - \frac{\alpha r}{1-b}) \times \dot{K} + \alpha \times \beta \times K = \alpha \times \beta \times r \times \frac{1}{1-b}$$

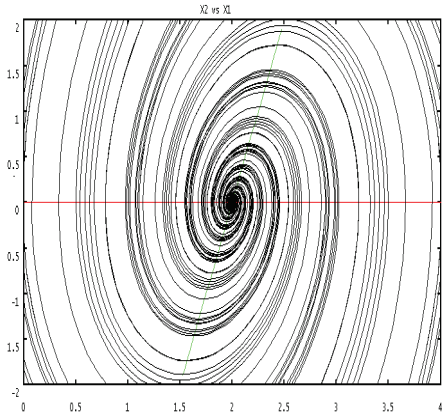
A particular solution can be found putting $K = \overline{K} = const.$, $\overline{K} = r \times \frac{a}{1-b} = r \times Y_e$ is the capital stock corresponding to the value Y_e of static equilibria. Using the relation $\dot{K} = I$ we obtain the dynamic system:

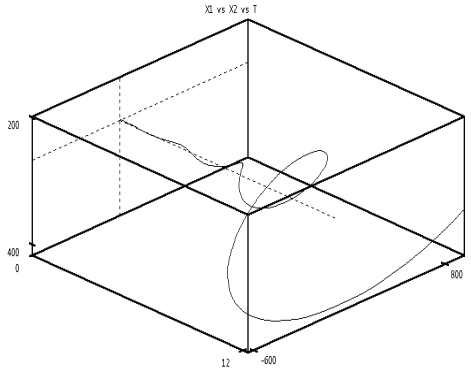
$$\begin{cases} \dot{K} = I, \\ \dot{I} = -\alpha \times \beta \times K - (\beta - \frac{\alpha \times \beta \times r}{1-b}) \times I + \alpha \times \beta \times r \times \frac{a}{1-b} \end{cases}.$$

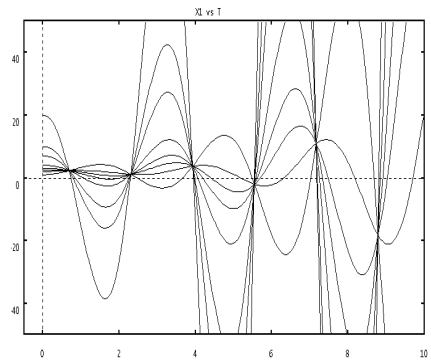
We note $c = \alpha\beta, d = \frac{r}{1-b}$ and obtain:

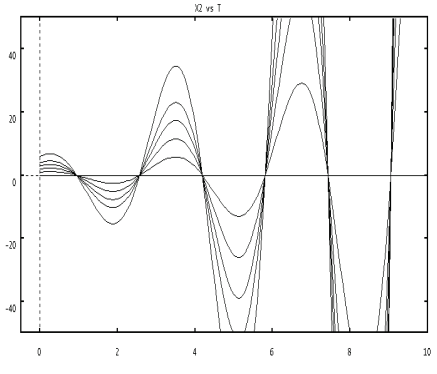
$$\begin{cases} \dot{K} = I, \\ \dot{I} = -c \times K - (\beta - c \times d) \times I + a \times c \times d \end{cases}.$$

For c and d fixed the equations $c = \alpha\beta, d = \frac{r}{1-b}$ define in R^5 o a three-dimensional sub variety. In all of its points the system develops identically, for every fixed value of it. This fact permits a big flexibility in choosing the more conveyable parameters, the designer having the ability to take in consideration the existent operations (stock, prices, and terms). In the situation in which $a = 2, \beta = 4, c = 4, d = 1$ the studied system admits a periodic solution, which means, in an economical matter, the apparition of a commercial cycle of business. This way the income growth is determined, the multiplication effect being produced. At its turn, the capital produces the accelerator effect over the investment. This thing is produced in the economic expansion phase, when K, I,Y









The evolution of the system from the equilibrium state to the explosive oscillation state increase ($a = 2, \beta = 3, c = 4, d = 1$).

4. CONCLUSIONS

Generally, with the activities being more complex, with the need of planning, search for strategies and formal actions grows. The economic domain is a domain in which the uncertain grade and risk is very high and in which the planning plays an important role in trying to reduce this incertitude. In essence, the elaboration of strategies in this domain purposes a clear and systematic structure of the modulations in which the followed objectives can be touched by a judicious allocation of the resources by long or short term. In the frame of any step of this type must be taken account of the most important aspects of the planning and certain the knowing the product description but also knowing the requests of the consumer because "the decisional act represents a compromise between the objectives and the restrictions."

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