MODELLING TECHNIQUES FOR LIFE PREMIUM RATINGS

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Abstract: The paper presents the modelling techniques used on international practice for establishing life premiums quota. Thus, the calculus techniques used by the insurers are generally based on a series of indicators named mortality indicators which mainly point out the insured persons' survival probability, the death probability and life expectancy at certain age. In Romania, these indicators are settled by National Institute of Statistics and they represent the basis for the calculation of the premiums quotes and for the elaboration by the insurers of premium tables. The benefit for the policyholder is to obtain insurance at a fair and competitive price and for the insurer, to maintain the experience of its portfolio in line with mortality assumptions.

1. Introduction

When speaking of life insurance, one should know that the insured person's rights and obligations are generally based on a series of indicators which mainly point out their probability of survival. These indicators are calculated by the National Commision for Statistics in each country and determine the insurance premiums quota. The most important role in determining the premium quota is played by the ***actuary,*** also named the life insurance mathematician.

In order to set the right premiums for the consumers, insurers use the experience studies which are based on informations about the past and the future. Usually, an experience study refers to mortality (for life insurance) or morbidity (for health insurance) experience. The knowledge and models obtained have a common element of mortality risk indicators but these are varied in different parts of the world. Experience studies help insurers minimise anti-selection (the exaggerated exclusion of some risks) and focus risk selection towards better and more profitable risk categories [Nabholz, 2007].

Life insurers quantify and group similar risks together, based on actuarially selection criteria. This criteria may not produce *equal* prices between insured persons, but it is *equitable* and guarantees that the consumer pays fair and competitive premiums based on the risk everyone brings to the group.

An experience study compares the actual experience on a block of business with a model of how the insurer anticipated that experience to look. This is referred to as an A/E ratio, where *A stands for actual*, and *E stands for expected* risks [JJ lane Caroll, 2007].

The very basic requirements to complete any type of mortality study include: policy issue date, issue age (or date of birth), gender, smoker status (dependent on the expected basis), policy face amount in force policy status (active, death, lapse, etc), termination date, claim settlement amount (if different from policy face amount), rating information about substandard risks, cause of death, and history of underwriting

guidelines and preferred criteria throughout the study period (published and internal to the insurance company).

Among these requirements, the most common *risk selection* factors that demonstrate sufficient credibility to form the basis of a study include gender, age at policy issue, smoker status, amount assured (as an indicator for socioeconomic status), and preferred class or underwriting rating. The latter two categories evaluate the combination of all underwriting that is performed on the group of lives.

It is also important to evaluate policy duration since issue. This is because risk factors tend to manifest themselves over time, resulting in a mortality curve with a selection period that eventually grades into ultimate mortality (i.e. the mortality expected at a given age in the absence of underwriting).

Thus, the life premiums are determined differentiate according with following factors: the indicators of the mortality tables; the frequency of payment of the premiums; the types of life insurance policies differing in terms of the covered risk (survival insurance, death insurance or mixed life insurance) and the way of paying the indemnity by the insurer.

2. Modelling techniques for premium ratings

The insurance industry has developed some models for an accurate calculation of premium ratings. The most acknowledged models are two types of them [Chessman Wekwete, 2007]:

- *multivariable models*, most important beeing *the Cox model*;
- *generalised linear models*.

In the abscence of the previous experience of the risks (statistical data), needful to transform these into premium ratings, there is used other tools, known as *multiple state models*, Markov model beeing one significant.

„The Cox model is a multivariable model used to analyse the effect of different risk factors on the time to an event" [Chessman Wekwete, 2007]. The risk factors (named covariates) used by Cox model are age, cholesterol level as continuous variables or gender and smoker status, as categorical variables. Various statistical software packages are available to calculate relative risks using different data which can produce an output that includes the estimates of the relative risks and their standard errors.

Generalised linear models – GLMs – allow for the analysis not only of survival times, but also other risk classification measures such as hazard rates, insurance claims exected over a specified duration, and proportions of diseased lives in a specified group of lives.

A significant constraint for Cox model and GLMs is that only risk factors for which values are available for all observations can be included in the model.

Markov models are defined by the specified states of insured lives and the hazard rates between the states. The *Markov property* is that the hazard rate coming aut of any state should depend only on the information that defines the state, and not on the history of the life prior to entering state.

Many-sided of Markov models is proved by the fact that premius can be calculated easily for any life insurance policy, on the condition that certain grounds are met: the states must be defined; all the hazard rates between the states must be defined, and must meet the Markov property criteria; the insurance payments related to these states and any other required information, such us inflation values or interest rates, must be provided.

Premiums are calculated by established formulae using mathematical software, and the premiums will reflect the risk associated with any set of risk factors.

For the planning of scientific rules in order to determine different calculation elements of the premiums is used the ***mortality tables indicators*** which are based on an international system of symbols as it follows:

- x – *age of the insured persons*;
- l(x) – *survival function* indicates how many persons belonging to an assumed generation of 100,000 living persons are still alive when attaining the age of *x* years;
- p(x) – *survival probability* expresses the chances of a person attaining the age of *x* years to continue to live till the age of *x+1* years;
- q(x) – *death probability* expresses the risk undertaken by a person who has already turned *x* years, that of dying before attaining the age of *x+1* years;
- d(x) – *number of persons supposed to pass away within x and (x+1) years* indicates how many persons of *x* years old passed away before the age of *x+1* years, being determined by the difference between the number of survivors aged *x* years (l_x) and the number of survivors aged *x+1* years (l_{x+1});
- E(x) – *hope of life at the age of x* or *life expectancy* represents the average number of years left to be lived for a person surviving the age of *x*;
- *Hope of life at birth* also named the span of life indicates the average number of years supposed to be lived by a newborn baby.

A common measure is the difference in the life expectancy between male and female. Life expectancy can be measured from any age, and is often measured from birth, but for assessing the impact of the mortality differential between males and females is appropriate to consider life expectancy in middle age [Devis, 1990].

For example, the researchers studies show the difference between life expectancy at age 45 for females and males in the EU Member States. Female life expectancy at age 45 significantly exceeds that of males in all EU countries. The difference ranges from 3.4 years (in Denmark) to 6.5 years (in France). Looking forward, in the UK Government's official national population projections (produced by the Government Actuary's Department), the sex differential in life expectancy at age 50 is expected to continue for many years in the future. From 3.6 years in 2002, it is expected to narrow only to 3.1 years by 2041 [Memorandum by Swiss Re, http://www.publications.parliament.uk].

For the life insurances, *these indicators represent the basis for the calculation of the net premiums* and for the elaboration of premium tables. In order to estimate the net premiums, the actuarial science provides general formulas for the estimation of these indicators, made up of symbols. A real determination of the premiums involves the subrogation (commutation) of one formula made of symbols with the figures corresponding to the insured person's age and to the interest which increase the premium. These figures meant to achieve the commutation of the formula are already calculated within the mortality tables for each age and they are called numbers of commutation. For the actualization factor (discount) marked with v^n, the percentage used by the insurance company in order to calculate the interest for the premiums, is considered and it accumulates in time.

For the estimation of the premium, the payment possibility are taken into consideration. For the life insurance, the premium to be paid is cashed once as a *unique premium* or *echeloned premiums.*

The *unique premium* is estimated in order to cover the risk during the whole insured period as the insurer cashes the total amount afferent to the insurance duration at the beginning of the contract. The cashed unique premium and the afferent interest resulting from its investment, will be used for the payment of the indemnity. This modality of payment is less used in practice, being applied for long term life insurances.

Insurances involving the *payments echeloned premiums* are often requested by the insured persons, because the amount representing the unique premium constitutes an important financial effort.

Another factor influencing the amount of the net premium is determined by the ***payment possibility used for the insurance indemnity and the type of the insurance policy*** (survival insurance, death insurance or mixed life insurance) and it is considered separately for each insured risk. Thus, in practice, there are the following situations considering the modality of paying the indemnity:

- The indemnity is completely paid in a certain number of years starting from the contract moment of the insurance policy (the unique payment of the indemnity).
- The indemnity is paid in installments as it follows:

- unlimited immediate annuities – the insured person pays the net premium at the conclusion of the insurance policy in order to receive the indemnity in installments (annuities), shortly after the conclusion of the contract (immediate annuities), during his whole life period, at the beginning or at the end of the year;

- limited immediate annuities - the insured person pays the net premium at the conclusion of the insurance policy in order to receive the indemnity in installments, shortly after the conclusion of the contract, for a limited period of time (limited immediate annuities);

- unlimited delayed annuities - the insured person pays the net premium at the conclusion of the insurance policy in order to receive the indemnity in installments (annuities), after a certain period of time from the conclusion of the insurance policy (delayed annuities), during his whole life period (delayed life annuities);

- limited delayed annuities - the insurer pays the indemnity in a certain number of years from the conclusion of the insurance policy (delayed annuities), but for a limited period of time (limited).

The *death insurance* relies on the premise that the insurer will pay the beneficiary of the insurance a certain amount of money at the date of death of the insured person. The determination of the unique net premium, in the case of death insurances, takes into consideration the contractual duration of the insurance which may be: undetermined (for life), a period of several years or a short period of time.

In practice, the *life insurance policy reflects more often a mixed nature,* thus, it covers the survival risk, as well as the death risk. Therefore, the insurer will pay the indemnity to the insured person, at the termination of the contract if the last one is alive, or the sum will be paid to the successors, at the date of death of the insured person. So, the unique net premium owed by the insured person is calculated by summing up the net shares of the premiums afferent to the two insured risks.

3. Formulas for life premium ratings

As we mentioned before, the net premium differs in accordance with the type on insurance policy, the insurance period and way of indemnity payment by the insurers. In the table 1, we synthesize the formulas for determining the unitary (at 1 c.u. indemnity)

unique (integral payment of premium by the insured person) net premium for the three basic life insurance types.

Table 1

The unitary unique net premium for survival, death and mixed life insurances

Survival Insurance		**Death Insurance**	
The payment of indemnity	**The unitary unique net premium**	**Insurance period**	**The unitary unique net premium**
The unique payment of the indemnity	${}_nE_x = \frac{D_{x+n}}{D_x}$	Death insurance one year term	$A_x = \frac{d_x}{l_x}$
Unlimited immediate annuities	$a_x = \frac{N_x}{D_x}$	Undetermined term	$A_x = \frac{M_x}{D_x}$
Limited immediate annuities	${}_{/n}a_x = \frac{N_x - N_{x+n}}{D_x}$	Limited immediate insurance	${}_{/n}A_x = \frac{M_x - M_{x+n}}{D_x}$
Unlimited delayed annuities	${}_{n/}a_x = \frac{N_{x+n}}{D_x}$	Unlimited delayed insurance	${}_{n/}A_x = \frac{M_{x+n}}{D_x}$
Limited delayed annuities	${}_{r/n}a_x = \frac{N_{x+r} - N_{x+r+n}}{D_x}$	Limited delayed insurance	${}_{r/n}A_x = \frac{M_{x+r} - M_{x+r+n}}{D_x}$
Mixed Life Insurance			
Insurance period		The unitary unique net premium	
The unique payment of the indemnity in case of survival risk and limited immediate valability for death risk		${}_{/n}AM_x = \frac{D_{x+n}}{D_x} + \frac{M_x - M_{x+n}}{D_x}$	

Notations used in the table have the following meaning:

- ${}_nE_x$ indicates the unitary unique net premium (the indemnity for a currency unit), due to be paid by the insured person aged x in order to receive the sum of 1 currency unit (c.u.) at the date of attaining the age of (x+n);
- a_x = unitary unique net premium paid by the insured person aged x for the insurance which provides him 1 c.u. during his whole life as indemnity. It results from reporting the total number of survivors aged over x years (N_x) to the number of living persons attaining the age of x years (D_x);
- ${}_{/n}a_x$ = unitary unique net premium paid by the insured person aged x for the insurance which provides him 1 c.u. in the next *n* years as indemnity. The difference from the numerator indicates the total number of survivors within the period of *x* and *x*+*n* years.
- ${}_{n/}a_x$ = unitary unique net premium paid by the insured person aged *x* for the insurance which provides him (if he is alive) 1 c.u. as indemnity, in *n* years, during his whole life;

- ${}_{r/n}a_x$ = unitary unique net premium paid by the insured person aged *x* for the insurance which provides him (if he is alive) 1 c.u. as indemnity, in *r* years for *n* years;
- ${}_{/n}AM_x$ = unitary unique net premium of the mixed insurance for a period of *n* years, by means of which the insurer pays the sum of 1 c.u. if the insured person is still alive in *n* years or the sum of 1 c.u. at the date of his death if it occurred before the moment *n*;

Dx, N_x and M_x are commutation numbers.

Number of commutation $D_x = v^x \cdot l_x$, where: v^n – life actualization factor or life discount factor ($v = \frac{1}{1+i}$, *i* indicates the actualization rate); *x* – age of the insured person; *n* – duration of the insurance policy.

Number of commutation $N_x = \sum_{k=x}^{\omega} D_k$ and it indicates the total number of the survivors after attaining the age of *x* years, for which the cashed premiums are capitalized using an actualization factor v^x. ω indicates the age of death of the last survivor.

Number of commutation $M_x = \sum_{k=x}^{\omega} C_k$, where $C_x = d_x \cdot v^x$. M_x expresses the total number of persons who will die after attaining the age of *x* years, considering a factor of capitalization v^x.

The values of D_x, N_x and M_x for each age become constant and they are listed in the tables of commutation numbers, elaborated considering the mortality table and the values assumed by the rate of actualization v^x.

For an indemnity, *S*, the total unique net premium (P) is obtained by multipling the unitary unique net premium specific to each type of insurance by *S* value.

Most frequently in practice, the insured person chooses the ***premium payment in annual, semestrial, quarterly or monthly installments***.

The *annual net premium* is determined by reporting the unique net premium to the annuity specific to the premium payment modality. Considering *r* years the duration of the premium payment, the annuity specific to the payment modality (a limited immediate annuity is considered), is given by the relation:

$${}_{/r}a_x^{(A)} = \frac{N_x - N_{x+r}}{D_x} \quad (1)$$

For example, for *survival insurance with unique payment of the indemnity*, annual net premium (p_x) is given by:

$$p_x = \frac{\frac{D_{x+n}}{D_x}}{\frac{N_x - N_{x+r}}{D_x}} = \frac{D_{x+n}}{N_x - N_{x+r}} \quad (2)$$

From the insurer's point of view, the life premium owed by the insured person is designed as the **gross premium** and consists of two elements: the net premium (or the basic rate) and the supplement or the extra premium.

The net premium serves to create the necessary fund for covering the indemnities or the insurance indemnities. The *determination of the net premium* takes into account the probability of risk occurrence and the intensity or the frequency of its manifestation. The *probability of risk occurrence* is given by the indicators of the mortality tables determined by the age of the insured person, in case of survival as well as in case of death. The *intensity of risk manifestation* is also given by the premium level, for risks of high intensity, the premium is also high, and for risks of low intensity, the premium is also low. If the risk has a variable manifestation during the contract, the premium will be modified in proportion to its intensity. The ***supplement or the extra premium*** covers the insurer's purchase and management overhead, as well as the ways of creating benefits. The value of these costs vary in terms of different types of insurance products and of different ways of dealing with them.

4. Case studies concerning premium fees for different types of insurance policies

4.1. Survival Insurance

One family made up of two persons aged 30, respectively 40 conclude a survival insurances for a period of 10 years and an insurance indemnity of 10,000 c.u., each one completely payable at the conclusion of the contract, if the insured persons are alive. The net premium is integrally paid at the conclusion of the insurance and the interest used by the insurer is of 30%.

Referring to the estimation report the unitary unique net premium specific to the survival insurance, for a unique payment of the insurance indemnity (Table no. 1) it results a total unique net premium, for the insured person aged 30, using the commutation numbers for 30%, which is determined as it follows:

$$P_{30} = S \cdot {}_{10}E_{30} = S \cdot \frac{D_{x+n}}{D_x} = S \cdot \frac{D_{40}}{D_{30}} = 10{,}000 \cdot \frac{2.523003}{35.986319} = 701.1\text{c.u}$$

So, for the insured person aged 30, in order to receive after 10 years the indemnity of 10,000 c.u., he has to pay as net premium, in the moment of contract policy 701.1 c.u. The numbers of commutation considered are selected from tables of numbers of commutation for the situation of Romania, both gender [Cristea, Dracea, Mitu, 2007].

If insured person is older, of 40 years, total net premium becomes:

$$P_{40} = S \cdot {}_{10}E_{40} = S \cdot \frac{D_{50}}{D_{40}} = 10{,}000 \cdot \frac{0.168916}{2.523003} = 669.5\text{ c.u}$$

We notice that, for an older age, net premium insurance is decreasing, because the insured person is older, the probability as insured person to be alive at the end of policy is reduced.

We presume that the insured person of 30 years chooses for the payment of net premium during o period of 5 years. Thus, annual net premium becomes:

$$P_x = S \cdot \frac{D_{x+n}}{N_x - N_{x+r}} = 10{,}000 \cdot \frac{D_{40}}{N_{30} - N_{35}} = 10{,}000 \cdot \frac{2.523003}{154.345 - 40.791} = 222 c.u.$$

So, for the insured person aged 30, in order to receive after 10 years the indemnity of 10,000 c.u., he has to pay as annual net premium, of 222 c.u., time of 5 years. Thus, he pays a total net premium of 1,110 c.u. (222 c.u. x 5 years), confronted by 701.1 c.u, in the case on integral payment of premium.

4.2. Mixed Life Insurance

An insured person aged 40 wishes to conclude a mixed life insurance on a period of 10 years, so that, at the age of 50 he may be returned an indemnity meant to provide him a certain standard of living. The insured person decides to pay maximum 1,000 c.u. annually, considering the value of the indemnity determined for both case, according to the table of commutation numbers: unique and annual payment of the premium, for a period of 5 years.

In order to determine the indemnity, one should take into account the estimation report between the annual net premium and the numbers of commutation from the table of commutation numbers, which corresponds to the age of 40 and for an annual interest of 20%.

$$p_x = S \times \frac{D_{x+n} + M_x - M_{x+n}}{N_x - N_{x+r}} \Rightarrow S = \frac{p_x(N_x - N_{x+r})}{D_{x+n} + M_x - M_{x+n}} = \frac{1{,}000(N_{40} - N_{45})}{D_{50} + M_{40} - M_{50}}$$

From number of commutation tabel for the 20% discount rate, there are selected the values of number of commutation, thus: N_{40}=356.61024, N_{45}= 136.27742, D_{50}=9.24219, M_{40}=2.56643 and M_{50}=0.75306.

Replacing in formula, we obtain:

$$S = \frac{1{,}000(356.61024 - 136.27742)}{9.24219 + 2.56643 - 0.75306} = \frac{1{,}000 \times 305.67545}{11.05556} = 19{,}930 c.u.$$

The insured person has the possibility of an unique payment of premium, determined as follows:

$$P = S\frac{D_{50} + M_{40} - M_{50}}{D_{40}} = 19{,}930\frac{9.24219 + 2.56643 - 0.75306}{62.00147} = 3{,}554 \text{c.u.}$$

If insured person will choose for integral payment of premium in the moment of contracting the policy, he will pay 3,554 c.u. for an indemnity of 19,930 c.u., and if he will choose for echeloned payments, he will pay, for the period of 5 years a total amount by 5,000 c.u. The supplement or the extra premium which represents insurer's expenses will be added at the net premium value and it wil be obtained the gross premium (total premium).

5. Conclusion

Practical experience of the around the world life insurers demonstrates that, they have to quantify and group similar risks together, based on actuarially selection criteria. The key finding of researchers' analysis is that, in all developed countries in the world, males have higher overall rates of mortality than females, even after allowing for other factors including whether or not a person smokes, their age and marital status, where they live and their lifestyle in general. Moreover, not only does female life expectancy exceed that of males today, but this has been the case in these countries for many years [Madigan, 1957].

By working with a diverse group of professionals such as statisticians, actuaries, doctors, underwriters, geneticists and other research professionals, it is possible to take a holistic approach to reviewing experience studies [Cristea, Criveanu, 2007]. The accumulated knowledge gained as pricing and underwriting environments change over time, in different parts of the world, can be employed to help insurers minimise anti-selection and focus risk selection towards better and more profitable risk categories. This ultimately leads to fairer and more competitive, and therefore more affordable, premiums for the end consumer.

REFERENCES

1. Carroll JJ. L. (2007) - „Evaluating risk with experience study analysis." Publication Swiss Re, *The Life risk selection at a fair price: reinforcing the actuarial basis*: 31-41;

2. Conn I., Graham (1987) - „Why Do Women Live Longer and Is It Worth It?", *British Medical Journal*, 294: 1691;

3. Cristea M., Dracea R., Mitu N.E. (2007) - *Life Insurance and Pension Funds*, Craiova, Universitaria Publishing House;

4. Cristea M., Criveanu R. (2007) - „Possibilities to Diversify Life Insurance in Romania", *Journal of Administrative Sciences* (JAS), 2007, Vol. 5, no. 1: 28-47;

5. Devis T. (1990) - „The expectation of life in England and Wales." *Population Trends* 60, 23-24;

6. Madigan C. (1957) - „Are Sex Mortality Differentials Biologically Caused?", *Millbank Memorial Fund Quarterly* 35(2): 202-33;

7. Mitu N. E., Giurca-Vasilescu L (2007) – "Tendințe de evoluție a serviciilor şi produselor în asigurări", *3rd International Scientific ECO-TREND 2006, Economics and Globalization*, November 24-25, 2006, Târgu-Jiu, Romania, Universitaria Publishing House, Craiova;

8. Nabholz C. (2007) - „The private insurance pricing and risk selection process", Publication Swiss Re, *The Life risk selection at a fair price: reinforcing the actuarial basis*: 9-17;

9. Office for National Statistics, ONS. (2002) - „Mortality statistics: general 2000" series DH1 no 33;

10. ONS. (2006) - „Mortality statistics: cause 2005" series DH2 no 32;

11. ONS. (2005) - „Mortality Statistics: General (England and Wales)" Annual review of the Registrar General on deaths in England and Wales, DH 1 No 38;

12. Pinkham C.A., Cumming M.E., Minuk H. (2006) - „The metabolic syndrome and all-cause mortality in an insured lives population." *North American Actuarial Journal*, Vol 10, No 3;

13. Ramey Estelle R. (2001) - „How Female and Male Biology Differ", *Women's Health Research: A Medical and Policy Primer*, ed. Florence P Haseltine. Washington, DC: Health Press International;

14. United Kingdom Parliament. Memorandum by Swiss Re, http://www.publications.parliament.uk/pa/ld200304/ldselect/ldeucom/165/4050507.htm

15. Wekwete C. (2007) - „Multivariable and multiple state modelling." *Publication Swiss Re, The Life risk selection at a fair price: reinforcing the actuarial basis*: 43-51;

16. Wingard D.L. (1982) - „The Sex Differential in Mortality Rates. Demographic and Behavioural Factors", *American Journal of Epidemiology*: 115, 206-216;

17. Woolnough K., Dickenson T. (2004) - „The sex mortality differential", *Swiss Re*.